OPERATIONAL ANALYSIS OF MARINE PROPULSION PLANTS

by

LIEUTENANT BARRY C. ROBERTS, U.S.C.G.

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

NAVAL ENGINEER

AND THE DEGREE OF

MASTER OF SCIENCE IN NAVAL ARCHITECTURE
AND MARINE ENGINEERING

at the

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ABSTRACT

The problem of applying the theory of systems analysis to the marine propulsion plant is investigated in this thesis. It is believed by the author that the use of this theory can be of considerable assistance to the naval engineer in giving him an understanding of the fundamental behavior of the system and will provide a scientific basis for design criteria and operational policies.

There are five basic areas of investigation covered in the thesis which all contribute to the organization of the overall method of accomplishing an operational analysis of a complex mechanical system. The following is a concise digest of each of the sections.

1. The derivations of the basic mathematical formulas for reliability determination are given. The concept of component failure distribution is presented and the exponential, normal, and Weibull failure models are discussed.

2. The importance of defining the objectives of the analysis clearly and completely is shown. The interpretation of component operating conditions and the resolving of success and failure standards are explained and an outline for the formulation of the necessary input data required to undertake the mathematical part of the analysis is offered. A hypothetical combined diesel-gas turbine propulsion plant is used to illustrate the proper procedures for performing a reliability analysis.

3. The mathematical procedures for determining system reliability by the correlation of the individual system components behavior are explained. In addition to presenting methods for solving serial, parallel, and Bayesian systems, formulas are derived for the reliability analysis of periodically operating components and of standby units. Representative values are postulated for the diesel-gas turbine plant and reliability curves are plotted which demonstrate the effect of component wear-out, reliability as a function of time, and the individual effect of specific components on the overall system reliability.

4. Two techniques for ascertaining maintenance schedules are formulated. The first technique yields the optimum time to perform preventive maintenance in order to maximize system availability or minimize expected repair costs. The second procedure deduces by dynamic programming an optimal decision policy on whether or not to repair the system dependent upon the number of trips made since the last repair and the number of trips still to be accomplished.

5. A brief investigation is made into the problem of components subject to the interaction of their failure distributions. Four possible interaction functions of instant failure rate are proposed and analyzed as to their effect on reliability and mean time to failure. An example is used to show how important this interaction effect can be when it is time dependent, such as would be caused by gradual wear.

In conclusion this thesis presents a procedure for analyzing a complex mechanical system using hypothetical examples to better illustrate the utility of the results to the naval engineer.

Thesis Supervisor: Ernst G. Frankel
Title: Assistant Professor of Naval Architecture

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The theoretical calculations in this work were done at the Computation Center at MIT, Cambridge, Massachusetts.

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NOMENCLATURE

Symbol	Meaning
$R(t)$	Reliability
$\rho(t)$	Instant failure rate
$f(t)$	Probability density function of failure
$F(t)$, $Q(t)$	Probability distribution of failure
T_f, MTBF	Mean time before failure
t, τ	Time
λ	Exponential instant failure rate
C	Constant

Specific nomenclature for diesel-gas turbine reliability analysis

Symbol	Meaning
C_1	$\rho(t)$ for diesel engine
C_2	$\rho(t)$ for gas turbine
C_3	$\rho(t)$ for heat exchange
C_4	$\rho(t)$ for reduction gear
C_5	$\rho(t)$ for lube oil pump
C_6	$\rho(t)$ for standby lube oil pump
C_7	$\rho(t)$ for fresh water pump
C_8	$\rho(t)$ for salt water pump
N	Mean time of non use of the gas turbine
C_2	Probability that the gas turbine will not be in use at time t

INTRODUCTION

Operational Analysis and Reliability Engineering

During and shortly after the Second World War, due to the growing complexity of all types of operations in general, a branch of mathematics termed operations analysis became prominent. Probability theory forms the basis for this field of science. If a definition is desired, it may be said the operations analysis is a scientific study of operations to better understand their behavior for the twofold purpose of predicting their future results due to changes in the system and controlling the operation to improve its result. This thesis is concerned with the use of operations analysis methods to solve problems in the field of reliability engineering.

In any system, mechanical or otherwise, one of the most important parameters determining its overall value is the reliability that the system can be expected to have during its operation. Although the exact definition of reliability is usually dependent upon who is the definer, the accepted definition endorsed by the American Society for Quality Control [1] states that reliability is the probability of a device operating within specified limits for the time and operating conditions imposed on the device. Operations analysis techniques are used in reliability engineering for the purpose of predicting reliability and optimizing the system from a reliability standpoint. While reliability analysis is a relatively new subject which has generally

only been applied to the electronics and missile field, it has the potential of being extremely useful in the understanding and improving of all types of mechanical systems including marine propulsion plants.

Importance of Reliability Analysis

The ability to be able to predict reliability is tantamount to understanding the underlying causes of unreliability. The consequences of unreliability are numerous; the most obvious ones being cost, lost time, and danger to personnel. The cost involved not only is that of the device to be replaced or repaired, but also the cost of the maintenance in material, manpower, and training with the added expense of having to continuously monitor the equipment for possible failures. The value of the analysis is that in choosing some specific system to perform a task, a reliability analysis of the system will furnish data on which to base a decision. For instance, the Armed Services mainly places the emphasis for selecting a component or system on the low bid price. Normally, reliability is only given token consideration in the decision. The question arises as to whether a reliability investigation of the device should be made since isn't it possible that it would be advisable to pay more initially for a reliable system that in the long run will give better availability and less maintenance costs than a cheap unreliable system? Conversely, the possibility arises as to whether in some applications too much reliability is required of a device. Certainly the adage that

"a chain is only as strong as its weakest link" applies in a system and, for example, designing a valve with a safety factor of 5 when the piping only has a safety factor of 2 is a waste of time and money. It is therefore evident that reliability apportionment should enter into any preliminary system design.

Another problem inherent to this day and age is that of complexity. The major problems to be solved have become more intricate as time passes and it is only natural that there is an increase in the complexity of the equipment designed to meet these problems. The more complex equipment is, the greater number of components are required, and the more likely it is that one of these components will fail. Thus, an increase in complexity leads to a decrease in reliability. Of what value is complex equipment with theoretically high performance if failures of its components keep the piece under repair most of the time. It can therefore be seen that engineering improvements for reliability will in many cases improve operability, maintainability and productivity with an overall net gain in system worth.

Other System Analysis Results

Results other than reliability may also be obtained from an operations analysis of a system. The question of maintainability of the system should normally be investigated unless for some special reason the system cannot be maintained during its operating cycle, i.e. missiles, torpedoes,

etc. A study of maintainability will give answers to such questions as;

1. Is preventive maintenance worthwhile for the particular system?

2. If worthwhile, at what time intervals should it be scheduled?

3. What is the optimum efficiency that can be expected from a system preventively maintained?

In conjunction with preventive maintenance scheduling, the renewal theory of operations analysis [2] will give the expected number of replacements for a component over its lifetime, t. This gives a good basis for making an estimate of the number of spare parts to be stocked.

Some other results that are obtainable are;

1. The effect of redundancy on reliability.

2. The system availability that can be expected.

3. Optimization procedures for reasons of reliability or cost.

4. Confidence levels that can be applied to the results of the analysis.

5. Information for the determination of equipment quality control specifications.

It is seen that by the methods of operations analysis, a deep insight into the behavior of a system can be obtained which will produce not only a better understanding of the system but also give a firm basis for making decisions to improve its overall value.

Present State of the Art

At the present time the state of the art of operational analysis of mechanical systems is very sketchy. Many mathematical models have been devised of hypothetical systems which have been analyzed thoroughly but the analysis of real systems has only been accomplished in the electronics and missile fields and even there only in limited situations. There are two main reasons for this lack of progress. First and foremost is the fact that the value of systems analysis and especially reliability engineering has just in the past few years come into prominence. Now that the mathematical tools have been developed and the value realized, the majority of large companies are beginning to organize reliability analysis departments. The second factor and possibly the hardest to solve is the cost and time involved in running the testing programs necessary for the analysis. In order to obtain any results from an analysis, failure tests must be run for the devices under investigation and reams of data must be evaluated. Although this statistical aspect of operations analysis will not be discussed in this thesis, it should be mentioned that there are sampling procedures that can be used effectively to make data correlation efficient. Inherently, the cost of initiating a reliability program will be high but just as inherently is the fact that in the long run, the program will more than pay for itself in its results.

Thesis Objective

The object of this thesis is to show how the methods of operations analysis can be employed in the investigation of a marine propulsion plant and of what worth such an investigation will be. The overall study of this question will be divided into five parts;

1. Reliability analysis basic concepts.

2. Formulation of the problem of determining system behavior by correlation of individual system component behavior.

3. Mathematical determination of system reliability.

4. Effect of preventive maintenance.

5. The problem of interacting components.

PROCEDURE

I. Reliability Analysis Basic Concepts

Reliability and Mean Time to Failure

Before introducing the method by which to solve the reliability analysis of a complex propulsion system, a short explanation of the theory behind the mathematical derivation of reliability will be given. This is deemed necessary since reliability analysis is essentially a new field and although it has as a foundation the mathematics of probability and of statistics, there are some intrinsic differences. The standard reliability terms are defined in the Nomenclature section of this thesis, however since the basic mathematical formulation is short, it will be included in the text.

To determine reliability mathematically, the following is presented;

Let

$x = f(x_1, x_2, \ldots, x_n)$ be a vector of performance characteristics of the component

A = region of the sample space representing the satisfactory performance of the component.

$w = f(w_1, w_2, \ldots, w_n)$
$z = f(z_1, z_2, \ldots, z_n)$ } be vectors specifying the upper and lower limits on satisfactory performance

t = time of the interval under investigation

$f(x,t)$ = probability density function of the performance characteristics

R = reliability of the component

therefore

$$R(x \subseteq A;\ t) = \text{Prob.}(w \leq x \leq z) = \int_A f(x,t)\ dx$$

employing the proper definitions [20]

$$R(t) = \int_t^\infty f(x)\ dx = 1 - \int_0^t f(x)\ dx = 1 - F(t)$$

furthermore

$$R(t) = e^{-\int_0^t \rho(x)\ dx}$$

where

$$\rho(t) = \frac{f(t)}{R(t)} = -\frac{d}{dt}(\ln R(t)).$$

$$f(t) = \rho(t)e^{-\int_0^t \rho(x)\ dx}$$

The other parameter of reliability mathematics that is of great importance is the components' mean time before failure (MTBF).

If T_f = mean time before failure

E = expected value

then

$$T_f = E(t) = \int_0^\infty t\ f(t)\ dt = \int_0^\infty t\ d\bar{R}(t) = \int_0^\infty R(t)\ dt$$

In the event that the component has been operating for a period previous to the time of investigation;

Defining $t = 0$ time component is new, or system starts operating

$t = t_1$ time of investigation

$t =$ time to point of interest

$R(t/t_1)$ = reliability of the component at time t given the component has operated to time t_1.

thus

$$R(t/t_1) = \frac{R(t)}{R(t_1)}$$

and

$$T_f = \int_{t_1}^{\infty} (t-t_1) \frac{R(t)}{R(t_1)} f(t) \, dt$$

$$= \frac{1}{R(t)} \int_{t_1}^{\infty} R(t) \, dt$$

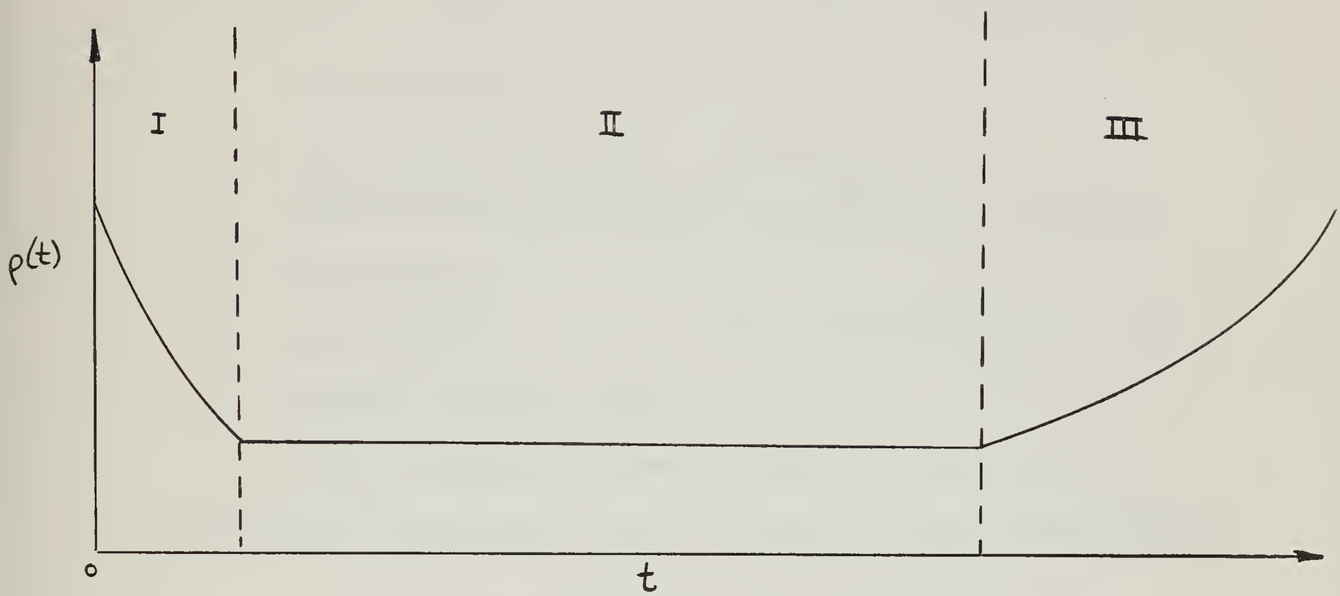
Component Failure Distributions

From the preceding paragraphs it can be seen that reliability and mean time to failure can be evaluated from the probability density life distribution of the component. In the following paragraphs some of the more important mathematical approximations of component life distributions are discussed. These distributions have been derived from extensive analysis of operating data and life tests. It must be remembered, however, that they are just statistical models characterizing a physical phenomenon and are, at least, a good approximation of the reality. For a better understanding of the real situation, confidence limits are associated with each of these approximations.

Normally, the characteristic parameter used to describe life distributions is the instant failure rate, $\rho(t)$. The time dependence of $\rho(t)$ that is representative of a normal component is shown in Fig. I.

FIGURE I

Representative $\rho(t)$ vs t plot



where

I = <u>early failure region</u> which results from manufacturing flaws, in-transit damage, etc. which cause normal failure at an early stage in the operating cycle. Usually in real situations there is a "debugging" or run-in period which eliminates this problem.

II = <u>chance failure region</u> caused by random failures which are normally considered independent of time and having a fixed probability of occuring at any time in this region.

III = <u>wear-out region</u> due to the gradual deterioration of the component with time.

<u>Exponential Distribution</u>

This is the most widely used life distribution in

reliability analyses. The reasons for this are it;

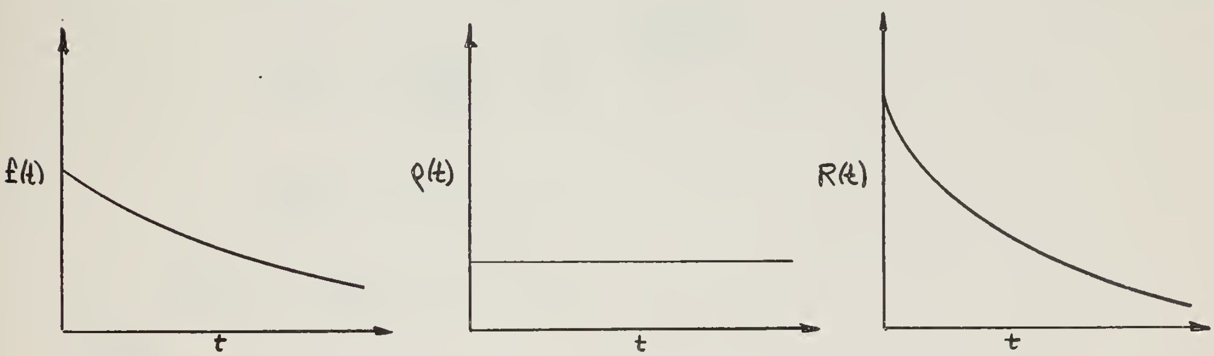
1. characterizes components that have been optimized to limit failures,
2. characterizes components having a predominence of human errors,
3. characterizes complex components,
4. characterizes components consisting of parts of mixed ages,
5. approximates what is usually the behavior in the chance failure region,
6. is the easiest mathematically to work with.

The equations associated with the exponential distribution are;

$$\rho(t) = \text{constant} = c$$
$$f(t) = \rho(t)\ e^{-\rho(t)t}$$
$$R(t) = e^{-\rho(t)t}$$
$$T_f = \frac{1}{\rho(t)}$$

FIGURE II

Exponential Distribution



Normal Distribution

This distribution is generally employed to describe the wear-out phenomenon. It also has been found to apply to;

1. components exhibiting homogeneous deterioration properties,
2. components subjected to small variations in environmental severity,
3. components whose failures occur at times well removed from $t = 0$, and whose MTBF is large compared to its standard deviation.

The equations associated with the normal (Gaussian) distribution are;

M = mean lifetime

σ = standard deviation of M

$$\rho(t) = \frac{e^{-\frac{(t-M)^2}{2\sigma^2}}}{\int_t^\infty e^{-\frac{(\tau-M)^2}{2\sigma^2}} d\tau}$$

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-M)^2}{2\sigma^2}}$$

$$R(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_t^\infty e^{-\frac{(\tau-M)^2}{2\sigma^2}} d\tau$$

$$T_f = M$$

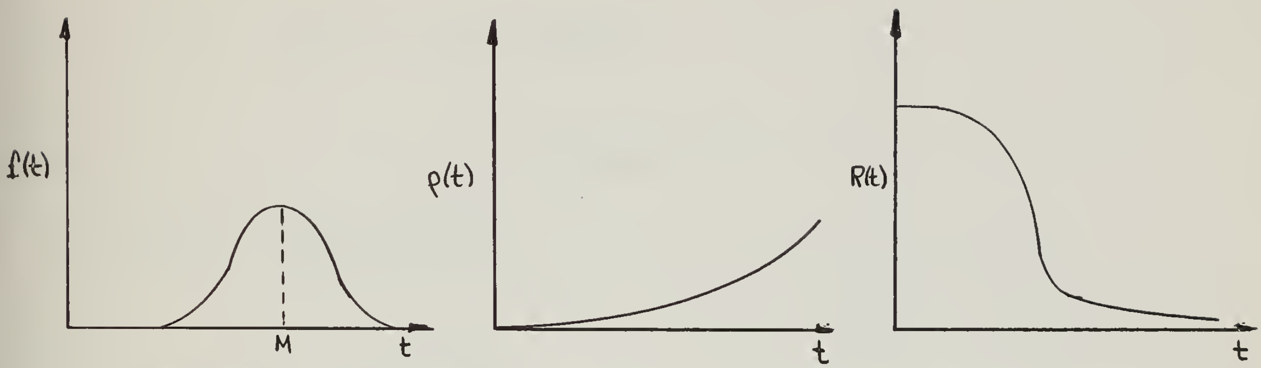
if $\operatorname{erf}(x) = \frac{2}{\pi} \int_0^x e^{-u^2} du$

then

$$R(t) = \frac{1}{2} [1 - \operatorname{erf}(\frac{t-M}{\sigma\sqrt{2}})]$$

FIGURE III

Normal Distribution



Weibull Distribution

Although this distribution has rarely been used in reliability analyses to date, statistical investigation of life tests of mechanical tests shows that this distribution seems to be applicable in many cases. The distribution can be described by a two parameter method or a three parameter method. It includes as a special case the exponential distribution and can approximate a normal distribution ($\beta = 3.25$).

For the three parameter case;

where α = scale parameter $\quad t \geq \gamma,\ \alpha,\ \beta > 0$

β = slope parameter

γ = location parameter

$$\rho(t) = \frac{\beta(t-\gamma)^{\beta-1}}{\alpha}$$

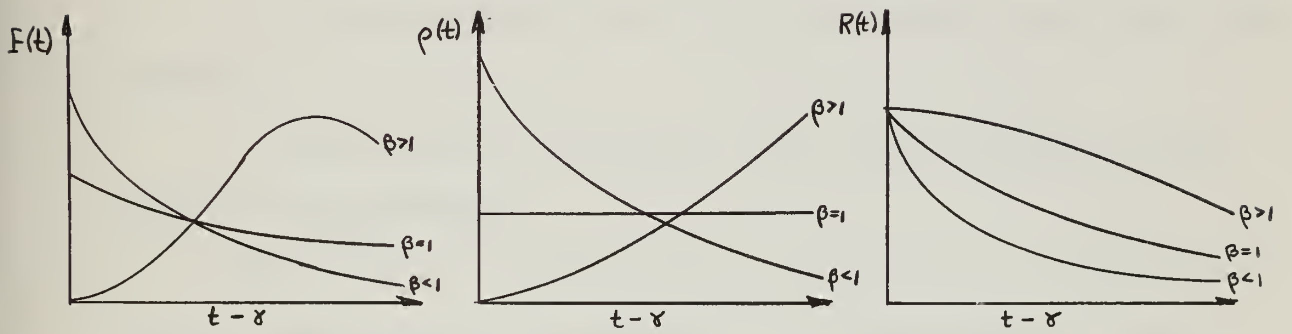
$$f(t) = \frac{\beta(t-\gamma)^{\beta-1}}{\alpha}\, e^{-\frac{(t-\gamma)^{\beta}}{\alpha}}$$

$$R(t) = e^{-\frac{(t-\gamma)^{\beta}}{\alpha}}$$

$$T_f = \gamma + \alpha^{\beta}\, \Gamma(\beta+1)$$

FIGURE IV

Weibull Distribution



Other Distributions

Some other distributions occasionally used in reliability studies are;

1. lognormal
2. gamma or beta
3. mixed Weibull

however, the exponential, normal, and Weibull are the most important of the distributions and are the ones used for examples in this thesis.

II Formulation of the Problem

Defining the Problem

One of the most critical parts of performing an analysis of any system is the complete definition of the problem to be studied. This definition basically must answer three questions;

1. What are the objectives of the analysis?

2. What components are to be considered comprising the system?

3. What will be considered the failure events of the system and components?

These must be clearly and exactly defined if clear and exact answers are to be expected. In some cases if just figure of merit or trend results are desired, greater liberties may be taken in the definitions. Once these three questions are answered, the problem has been defined and the results of the investigation are completely dependent upon these definitions.

Objectives

When defining the objective of the analysis, the question is, what is wanted and what should be expected from the analysis. In order to better illustrate how to accomplish an operations analysis of a system, a hypothetical marine propulsion plant will be postulated and investigated. It must be realized by the reader that this example will be necessarily simplified since the object is not numerical results but instead the method of the solution.

The objective of this analysis will be to find the reliability of a combined diesel and gas turbine marine propulsion plant. The result desired is how does the reliability of the overall plant vary with time in operation and how should one go about improving this reliability. Naturally in this example the value of the numerical results are useless since there is no statistical basis for the numbers used. In an actual analysis, statistical life tests would have to be performed on all the components involved. From these statistical tests, confidence limits can be found and applied to the final results. Methods of finding confidence levels are discussed in Lloyd and Lipow [3].

System

The next step in the problem is to define the system. The system can be defined in as basic or complex terms as the investigator deems necessary in order to obtain the results desired. In this example the system will be comprised of the following components;

1. diesel engine
2. lube oil pump
3. fresh water pump
4. salt water pump
5. heat exchanger
6. gas turbine
7. reduction gear
8. standby lube oil pump.

From the definition of reliability it is seen that in defining the system, the operating conditions which are within the prescribed limits must also be enumerated. There are five basic operating conditions that should be taken into account;

1. environmental
2. conditions imposed by the operator or user of the system
3. functional dependence of the components within the defined system
4. interaction of component failure distributions
5. maintenance policies.

It is altogether possible that more than one combination of these conditions may want to be investigated. In this case different reliabilities can be expected for the different combinations. For the purpose of this investigation the following operating conditions are defined;

Environmental: The only extraordinary environmental conditions imposed on the system will be the possibility of shock with the probability of P_s of occurring.

Imposed conditions: The system must be able to operate for 20% of its total operating cycle above cruising power.

Functional dependence: The functional dependence of system operation upon the components is;

1. Diesel must perform successfully for the system to operate.
2. Reduction gear must perform successfully for the

system to operate.

3. Heat exchanger must perform successfully for the system to operate.
4. Salt water pump must perform successfully for the heat exchanger to operate.
5. Fresh water pump must perform successfully for the heat exchanger to operate.
6. Gas turbine must perform successfully when more than cruising power is required.
7. Lube oil has to be available to the diesel at all times.
8. A defined shock level with a probability of P_s of occurring will cause failure of the reduction gear and heat exchanger.

Interaction effects: No interaction among the component failure distributions will be assumed. This is not necessarily the case in the real system but at the present state of the art, interaction effects cannot be handled except by using "guess" factors. In a later part of the thesis, these interaction effects will be investigated.

Maintenance policies: For the propulsion plant of this example, no true maintenance actions will be considered performed. It will only be assumed that minor adjustments for power, temperature, and fuel regulation will be made. Of course in most systems, maintenance actions will improve the reliability and this effect will be

covered in another section of the thesis.

Concept of Success and Failure

In order to determine the failure distribution of a component, it is necessary to define what is considered a failure and what a success in the operation of the component. It is not necessarily true that the only failure of a component is when it stops functioning because in many situations, the system will also fail if a component operates outside certain tolerances. For instance, if a lube oil pump doesn't supply the oil at the required pressure even though the pump may still be running, the system will fail. It is therefore evident that the investigator must be careful in choosing the ground rules for success or failure of any test and relate them to the various modes of operation.

To find the failure distributions relating to the components, a four step procedure must be followed;

1. Define success and failure of the entire system being analyzed for a particular mode of operation.

2. Relate the definition determined from 1 to each individual components operation so that success-failure criteria may be ascertained for each component.

3. Establish the type of statistical distribution which describes the failure phenomena.

4. Estimate the parameters which completely define these distributions.

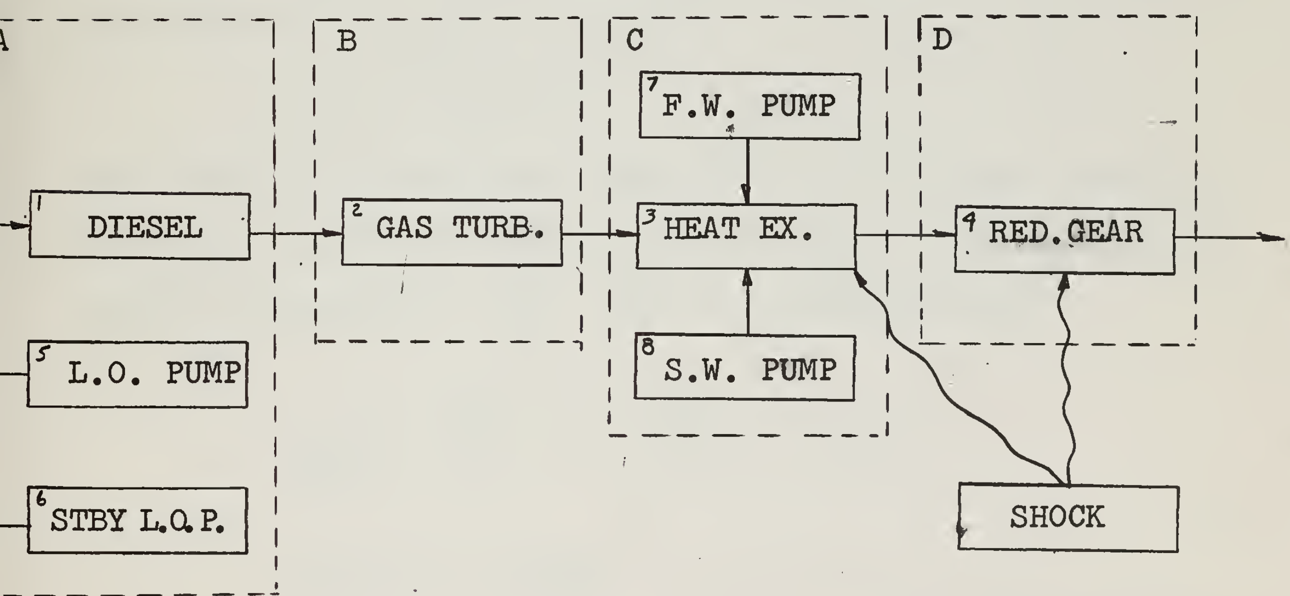
After this is accomplished, the investigator is ready to begin the mathematical aspect of the analysis since all input data are now available. For the example of this thesis, it will be assumed that the above steps have been carried out

and the failure distributions of the components in question are known. The case where the out of tolerance distribution is independent of and different than the "catastrophic" failure distribution will be taken into account in one case.

Reliability Schematic Block Diagram

The reliability schematic block diagram sometimes called the reliability structure model is a picture form of the functional relationship of the system and the components under investigation. It is not a necessary facet of an operational analysis, however in complex situations the model will provide an insight into how to best attack the problem from the mathematical standpoint. From this diagram it is also easier to recognize the weaknesses or potential weaknesses of the system from a reliability sense. The reliability schematic block diagram for the thesis example is shown in figure (V).

FIGURE V



III Mathematical Procedure

Three Basic Rules of Reliability Mathematics

Before the mathematical analysis of the system is begun, a statement of the basic reliability rules of probability calculus will be discussed.

The first rule states that if there are N mutually independent components, each having a reliability of $R_i(t)$ and that for satisfactory operation of the system all the components must function properly; then the overall system reliability $R_{sys}(t)$ is

$$R_{sys}(t) = \prod_{i=1}^{N} R_i(t)$$

This is known as the Product Rule of Reliability and a system of this type is called a serial system.

The second rule states that if there are N mutually independent components each having a reliability of $R_i(t)$ and unreliability of $Q_i(t)$ so that

$$R_i(t) + Q_i(t) = 1$$

and that for satisfactory operation of the system, only one of these components must function properly, then the overall system reliability $R_{sys}(t)$ can be determined since

$$Q_{sys}(t) = \prod_{i=1}^{N} Q_i(t) = \prod_{i=1}^{N} (1-R_i(t))$$

$$R_{sys}(t) = 1 - Q_{sys}(t)$$

$$R_{sys}(t) = 1 - \prod_{i=1}^{N} (1-R_i(t))$$

This is known as the Product Rule of Unreliabilities and a system of this type is called a parallel system. This is usually described by the term redundancy, relating to the fact that there are alternate components to help the system operate successfully in case of failure of one or more of the other components. The normal method for increasing system reliability is to add redundancy to the original system.

The third and most important rule is based on Bayes' probability theorem. It states that the probability of system failure is the probability of system failure given that the component is bad times the probability the component is bad plus the probability of system failure given that the component is good times the probability the component is good.

$$Q_s(t) = Q_s(t)|R_i(t) \times R_i(t) + Q_s(t)|Q_i(t) \times Q_i(t)$$

This is called the Conditional Rule of Reliability and can be applied to any system including serial and parallel.

Mathematical Analysis of the Complex System

With these rules the analysis of the complex system can be begun. They will not account for every situation encountered in the investigation but they will be the foundation for all the computational work done.

For the purpose of simplicity, the components will be referred to by the number in the upper left hand side of the block in Figure V, i.e. 1 = diesel, 2 = gas turbine, etc. It is seen that blocks A, B, C, D form a serial system such

that if any one of these fail the system fails. If the reliability of each of these four blocks could be found then

$$R_{sys}(t) = R_A(t) \cdot R_B(t) \cdot R_C(t) \cdot R_D(t) \qquad \text{I}$$

This will be the approach to determ ining the overall system reliability.

Reduction Gear

The first component to be examined will be the reduction gear. It is known that the reliability of the gear is $R_4(t)$ under normal operating conditions. However, it is also known that some predefined shock amplitude will cause the gear to fail.

Define P_s as the probability that a shock of the defined amplitude will occur

$$Q_s = 1 - P_s$$

Reliability of D = Reliability of 4 given no shock occurs times the probability of no shock plus the reliability of 4 given the shock occurs times the probability that shock occurs.

$$R_D(t) = (R_4(t) | P_s = 0)\ Q_s + (R_4(t) | P_s) P_s$$

$$(R_4(t) | P_s = 0) = R_4(t) \qquad R_4(t) | P_s = 0$$

therefore

$$\underline{R_D(t) = R_4(t)\ Q_s} \qquad \text{II}$$

Heat Exchanger

In finding the total reliability of block C, we again use the Bayes Theory approach of Rule 3.

$$R_C(t)(P_s = 0) = R_3(t)(\text{if 7 is good}) \cdot R_7(t) + R_3(t)(\text{if 7 is bad}) \cdot Q_7(t)$$

$$= R_3(t)(\text{if 7 is good}) \cdot R_7(t)$$

as $R_3(t)(\text{if 7 is bad}) = 0$

which leaves $R_3(t)(\text{if 7 is good})$ to be found

$$R_3(t)(\text{if 7 is good}) = R_3(t)(\text{if 8 is good}) \cdot R_8(t) + R_3(t)(\text{if 8 is bad}) \cdot Q_8(t)$$

$$R_3(t)(\text{if 7 is good}) = R_3(t)(\text{if 8 is good}) \cdot R_8(t)$$

as $R_3(t)(\text{if 8 is bad}) = 0$

but

$$R_3(t)(\text{if 7 and 8 are good}) = R_C(t)(P_s = 0)$$

$$\therefore R_C(t)(P_s = 0) = R_3(t) \cdot R_7(t) \cdot R_8(t)$$

and it is seen that the dependence of the heat exchanger on the fresh water and salt water pumps is the same as a subsystem composed of these three components in a serial arrangement. However, the shock hasn't been taken into account therefore

$$R_C(t) = R_3(t) \cdot R_7(t) \cdot R_8(t) \cdot Q_s \qquad \text{III}$$

Gas Turbine

In the analysis of the block containing the gas turbine the first unusual case of mathematical reliability prediction appears where the three basic rules are not sufficient enough to give an answer. In this case, probability theory must be used to formulate a new type of reliability equation. This is due to the fact that the gas turbine is only used periodically and only needs to operate properly when called into use.

The reliability of the gas turbine can therefore be found by the following procedure;

Define

$f_2(t)$ = failure density function of the gas turbine

$P_2(t)$ = probability that the gas turbine is not in use at time $\underline{t}$

$U_2(t)$ = distribution function of periods of non-use of the gas turbine

$Q_2(t)$ = unreliability of the gas turbine.

The situation can be described by the statement; a failure of the gas turbine at time $\tau \leq t$ causes system failure by $\underline{t}$ under one of two conditions:

1. The gas turbine is in use at time τ when it fails.
2. The gasturbine is not in use at τ but is called into use after τ and at or before t.

Condition $\underline{1}$ is characterized by

$$Q_2'(t) = \int_0^t [1-P_2(\tau)]f_2(\tau)d\tau$$

Condition $\underline{2}$ is

$$Q_2''(t) = \int_0^t P_2(\tau)U_2(t-\tau)f_2(\tau)d\tau$$

and

$$Q_B(t) = Q_2'(t) + Q_2''(t) \qquad \text{IV}$$

Since the periods of non use are considered to be randomly spaced.

Defining

N_2 = mean period of gas turbine non-use

the non-use density function is

$$u_2(t) = \frac{1}{N_2} e^{-t/N_2}$$

and

$$U_2(t) = \int_0^t \frac{1}{N_2} e^{-\tau/N_2} d\tau = \left[- e^{-\tau/N_2} \right]_0^t$$

$$U_2(t) = 1 - e^{-t/N_2}.$$

Diesel Engine

In deriving the reliability equation for block A, encompassing the diesel engine, another situation arises when the three basic reliability equations are not adequate. This situation is caused by the lube oil-standby lube oil pump combination and this combination is referred to as a standby system. To find the reliability of such a system

Define

$f_5(t)$ = failure density function of the lube oil pump

$f_6(t)$ = failure density function of the emergency lube oil pump

Since the standby lube oil pump only operates at some time t_1, when the lube oil pump fails, the true failure density functions for the system are

$f_5(t_1)$ lube oil pump

$f_6(t-t_1)$ = standby lube oil pump

As the only time the system fails is when both pumps fail, the system unreliability is found by Rule 2.

$$Q_{standby}(t) = Q_5(t)Q_6(t) = \int_0^t f_5(t_1)dt_1 \int_0^t f_6(\tau-t_1)d\tau$$

and the failure distribution function of the emergency lube oil pump

$$F_6(t-t_1) = \int_{t_1}^{t} f_6(\tau-t_1)d\tau$$

$$R(t) = 1 - Q(t)$$

consequently

$$R_{standby}(t) = 1 - \int_{0}^{t} F_6(t-t_1)f_5(t_1)dt_1. \qquad \text{V}$$

Using the same type analysis as for the heat exchanger

$$R_A(t) = R_1(t)R_{standby}(t) \qquad \text{VI}$$

Overall System Reliability

To find the overall system reliability, equation I is applied

$$R_{sys}(t) = R_A(t)\cdot R_B(t)\cdot R_C(t)\cdot R_D(t) \qquad \text{I(a)}$$

where

$$R_A(t) = R_1(t)\cdot R_{standby}(t)$$

$$R_B(t) = [1 - Q_2(t)]$$

$$R_C(t) = R_3(t)\cdot R_7(t)\cdot R_8(t)\cdot Q_s$$

$$R_D(t) = R_4(t)\cdot Q_s$$

Equation I(a) is applicable for the overall system reliability for any type failure distributions for the components i.e. exponential, normal, Weibull, etc. In most instances, however, the solution of such complex equations necessitates the use of a digital computer. For the purpose of easy understanding and readily obtainable results, the failure distributions of the components will be assumed exponential. This is the standard method in the majority of reliability analysis and for most complex systems not too unrealistic, at least for a first approximation. The

assumption of the exponential distribution requires two basic postulates;

1. The components are not operated long enough so that they begin to wear out due to aging. From a reliability standpoint this is a logical way to operate the components as their reliability greatly decreases as they enter the wear out regime.

2. Any failure of a component is completely random, i.e. it is not time dependent.

Using the exponential distribution, the following equations are arrived at;

$$R_A(t) = e^{-C_1 t}[e^{-C_5 t} + \frac{C_5}{C_6 - C_5}(e^{-C_5 t} - e^{-C_6 t})]$$

See Appendix [B] for proof.

$$R_B(t) = e^{-C_2 t}[1 + P_2(t)(\frac{C_2}{\frac{1}{N_2} - C_2})] + e^{-\frac{t}{N_2}}[P_2(t)(\frac{C_2}{\frac{1}{N_2} - C_2})]$$

See Appendix [B] for proof.

$$R_C(t) = Q_s[e^{-(C_3+C_7+C_8)t}]$$

$$R_D(t) = Q_s[e^{-C_4 t}]$$

thus

$$R_{sys}(t) = Q_s^2\, e^{-(C_1+C_3+C_4+C_7+C_8)t}\left[e^{-C_5 t}\left(1+\frac{C_5}{C_6-C_5}\right)-\left(\frac{C_5}{C_6-C_5}\right)e^{-C_6 t}\right]$$

$$\text{times}\ \left\{\left[e^{-C_2 t}\left(1+\frac{P_2(t)C_2}{\frac{1}{N_2}-C_2}\right)\right] + e^{-\frac{t}{N_2}}\left[\frac{P_2(t)C_2}{\frac{1}{N_2}-C_2}\right]\right\}$$

Quantitative Analysis

In order to more graphically show the value of the reliability analysis, representative values for the component instant failure rates $(\rho_2(t) = C_i)$ will be chosen. The overall system reliability as a function of time can then be evaluated.

The values chosen are;

$C_1 = .0002 \quad hr^{-1}$

$C_2 = .0003 \quad hr^{-1}$

$C_3 = .00006 \quad hr^{-1}$

$C_4 = .00004 \quad hr^{-1}$

$C_5 = .0004 \quad hr^{-1}$

$C_6 = .0005 \quad hr^{-1}$

$C_7 = .0005 \quad hr^{-1}$

$C_8 = .0005 \quad hr^{-1}$

$Q_s = .98$

$P(t) = .80$

$N_2 = 200 \quad hr$

Also, assume the wear-out of only one component is taken into account. This component's failure due to wear-out shall

be described by a normal distribution with a mean life time of 1200 hrs and a varience of 400 hrs.

Figure VI shows $R_{sys}(t)$ vs t for the cases with and without one component wear-out consideration.

For the purpose of understanding the individual components relationship to the system as a whole, each of four component failure distributions have been varied to find its total effect on the system. These four components were chosen due to the different nature of their operation;

Component	Time Operation
Diesel	continuous, dependent upon L. O. system
Gas turbine	periodic
L. O. pump	continuous with standby system
F. W. pump	continuous, nondependent

IV Preventive Maintenance:

In most of the reliability work done in the past, little interest has been given to the effect of preventive maintenance . The reason for this is twofold due to the fact that most of the reliability study has been applied to the missile and electronic fields;

1. In the missile field the operation is usually "one shot" i.e. the component or system having once begun its operation cannot be maintained, and after terminating this operation it is unusable.

2. In the electronic field, the failure distributions of the components are considered exponential thus making preventive maintenance useless. This is so because the instant failure rate is independent of time and therefore replacing the component at some specific time when it is still operating successfully has no advantageous effect from a reliability standpoint.

These two conditions do not necessarily hold however, for a marine propulsion plant. Such plants definitely are not "one-mission" devices and it is doubtful if the exponential distribution will be applicable to the mechanical components found in such systems. It is proven by R. F. Drenick [4] that if the instant failure rate is constant or decreases with time, then preventive maintenance is not beneficial. In mechanical components, it has been determined that the instant failure rate normally increases with time. In this case preventive maintenance can be worthwhile and should be considered

in any operational analysis of a marine propulsion system.

A maintenance investigation should provide two basic parameters; how valuable to system performance is preventive maintenance, and at what time intervals should it be performed. For the purpose of analysis two different replacement policies will be explored. The mathematical analysis is from the work of R. Barlow and L. Hunter [5].

Policy I:

After every t_o hours of continuing operation without failure, preventive maintenance is performed. If there is a failure before t_o, the maintenance is performed at this time, t_1, and the next maintenance is scheduled at time $t_1 + t_o$.

Defining

η_∞ = expected fractional amount of time the system is operating as $t \to \infty$

the two parameters that determine the condition of the problem are η_∞ and t_o. The criterion of optimality will be the maximization of η_∞.

T_e = expected time needed to perform repair to the system after a failure

T_s = expected time needed to perform a scheduled preventive maintenance action.

By making the assumption that after each repair action, the system is as good as new, the following equations are applicable for optimal results;

$$\rho(t_o) \int_0^{t_o} R(t)dt - F(t_o) = \frac{T_s}{T_e - T_s} \qquad T_e > T_s$$

$$\eta_{1\infty} = \frac{1}{1+(T_e-T_s)\rho(t_o)}$$

if $T_e < T_s$, no preventive maintenance should be made and

$$\eta_{1\infty} = \frac{T_f}{T_f+T_e}$$

<u>Policy II</u>:

In this case, preventive maintenance is performed on the system after it has been operating for a period of t^o hours regardless of the number of intervening failures. If there is a failure before t^o hours have elapsed only minimal repair is made which in effect puts the system back in operating condition but does not change the basic system failure rate.

Using the same definitions for η_∞ and T_s and defining;

T_m = expected time needed to perform minimal repair to the system

the following equations are determined

$$\int_0^{t^o} t\ \rho'(t)dt = \frac{T_s}{T_m}$$

$$\eta_{2\infty} = \frac{1}{1+T_m\ \rho(t^o)}$$

The question naturally arises as to which policy should be used for a particular situation. This question can be resolved by equating $\eta_{1\infty} = \eta_{2\infty}$ for a given T_s which is known and the same for both policies. By substitution of

values for T_e, T_m can be determined. By graphing T_e vs T_m for a given T_s, which policy to use becomes evident. Figure XI shows such a graph for a Weibull distribution.

A Maintenance Policy by Dynamic Programming

Assume that a preventive maintenance policy is to be determined for a system having an instant failure rate that is a linear function of its operating time. The system is to be operated in discrete time intervals of length t_o. If the system is repaired between operating periods the cost associated with the repair is C_r. If the system fails during an operating interval it cost C_f for the repair and the system cannot be operated again till the next interval. Define

C_r = cost of scheduled maintenance

C_f = cost of "in service" repair

P_m = Probability of failure in interval $m + 1$ given it has not failed in m intervals since the last maintenance

$f_n(m)$ = least expected cost of making n more intervals given m intervals have been made.

There are two policies that can be followed, either repair the system at m or do not repair it. By the methods of dynamic programming covered by R. Bellman [6] the following dynamic programming equation is developed;

$$f_n(m) = \min \begin{cases} R: \; C_r + P_o[C_f + f_{n-1}(0)] + (1-P_o)f_{n-1}(1); & \underline{\text{repair}} \\ NR: \; P_m[C_f + f_{n-1}(0)] + (1-P_m)f_{n-1}(m+1); & \underline{\text{non-repair}} \end{cases}$$

$C_f > C_r$ for logical results otherwise preventive maintenance is not worthwhile.

To show mathematically the effect of the failure repair cost, C_f, and the scheduled maintenance cost, C_r, on the maintenance policy to be followed, the following problem will be postulated.

A system has a failure distribution curve that can be described by the Weibull approximation with the following parameters;

γ = location parameter = 0

β = shape parameter = 2

α = scale parameter = 57×10^4 hrs

$\alpha^{1/\beta}$ = characteristic lifetime = η = 755 hr

$M(t) = N(t)$ = interval time = 100 hr

therefore

$$\rho(t) = \frac{2(t)}{57(10^4)}$$

$$R(t) = e^{-\frac{t^2}{57(10^4)}}$$

The question to be answered is how does the maintenance policy vary as a function of m, n, C_f, and C_r. Figures XII through XVI give the results of this investigation.

V Interaction of Component Failure Distributions

In the previous sections of the thesis, the overall reliability of a hypothetical complex marine propulsion plant has been derived by the proper mathematical combination of individual component reliabilities. In actual situations however, it has been found that although this method is mathematically correct, it does not yield the actual reliability observed in practice. Intuitively, it might at first glance appear that this poor correlation is because the model is not a good functional representation of the real system. Although this is a possible reason, further analysis may show that mathematically all work is accurate. Where, then, is the mistake in the analysis? One of the major possible explanations and one that although recognized is not included in present reliability studies, is that when components are integrated into the system, there is an interaction among them which can change their failure distribution. Because of this, the failure distribution observed in the single component life test is not applicable and must be modified. The problem of finding what type of function this modification factor is, will in actual practice be very difficult. However, if reliability investigations are to be accurate, component failure rate interaction has to be taken into account as it may have a pronounced effect on system reliability, preventive maintenance policies, and system availability calculations. It is therefore imperative that laboratory life tests of components suspected of interaction and data assimilation of real system

operation be analyzed so that interaction functions can be determined.

For the purpose of illustration, two possible situations where interaction may play a substantial role in an investigation are given.

a. The failure distribution of a diesel cylinder liner will be affected by the wear of the piston rings, the wear of the connecting rod bearings, and the fuel quality.

b. The failure distribution of a reduction gear will be dependent upon the wear of the shaft bearings and coupling operation.

Since no data is available on interaction, for the purpose of investigation only hypothetical interaction functions can be postulated. In the following sections such assumed functions are examined.

<u>Effect of Interaction on Component Reliability</u>

The time dependent reliability of a component is described by

$$R(t) = \exp[-\int_0^t \rho(\tau)\, d\tau]$$

which is a function of a single variable, $\rho(t)$ = the instant failure rate. An equation illustrating the main factors that influence component failure rate is characterized by

$$\rho(t) = f(\alpha, \beta, \phi, t)$$

α = derating factor

β = environmental stress factor

ϕ = interaction factor

t = operating time

where

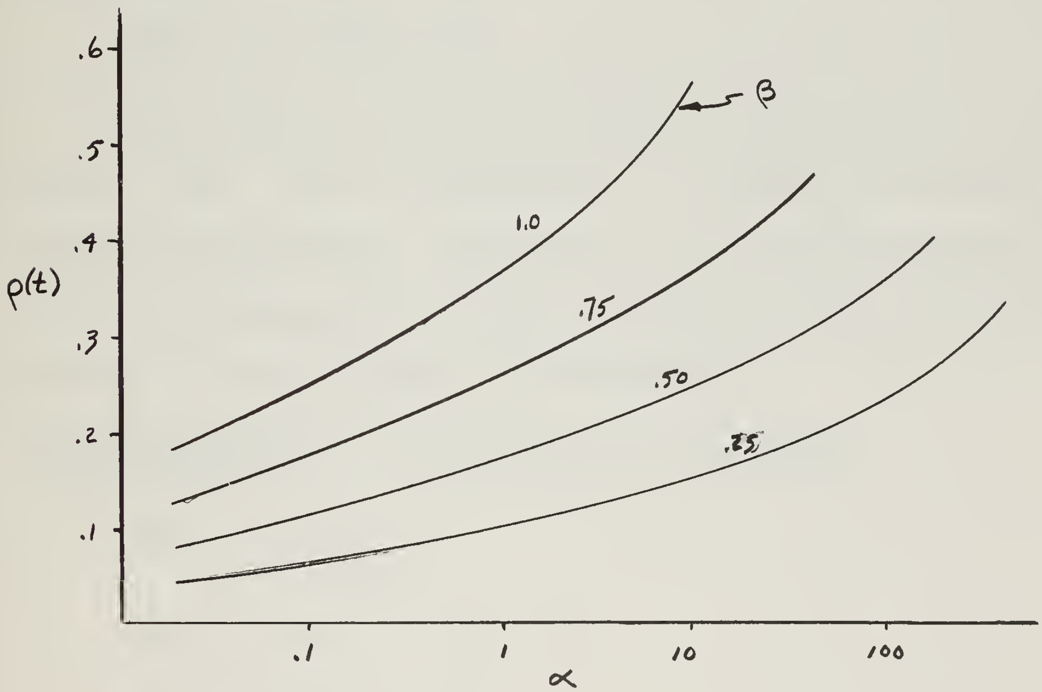
α = accounts for the fact that the component can operate at different outputs

β = is a function of the operational environment the component experiences, i.e. temperature, vibration, and acceleration stress levels

ϕ = accounts for the effect of interaction among the component failure distributions

t = time the component is in operation

It is normally possible to find the instant failure rate as a function of derating factor, stress factor, and time from experimental life testing of the component. An example of such a function can be shown in a diagram for a component that follows the exponential failure distribution.



Because reliability engineering is relatively new, little experimental data is available to find such functional relationships and what is available only applies to the exponential failure distribution. It is valid to assume that the same type of relationships or equations can be found for other types of components and failure dsitributions. The problem still presents itself ; how can the interaction factor be handled? Since no work has been done on this matter, only conjectural relationships can be assumed for the purposes of investigation. Such relationships will therefore be assumed and their properties analyzed.

Define

$\lambda(t)$ = component instant failure rate without taking into account this interaction factor, ϕ.

therefore

$$\rho(t) = f\,(\,\lambda(t),\ \phi)$$

It is this function that will be hypothesized. In all cases $\lambda(t)$ will be considered a constant for ease of mathematical computation. The effect of the interaction factor which is determined in this exponential case will apply in effect to other failure distributions.

<u>Investigation 1</u>: ϕ = additive and constant

$$\rho(t) = \lambda + \sum_n C_i$$

$$R(t) = \exp[-(\lambda + \sum_n C_i)t]$$

$$T_f = 1/(\lambda + \sum_n C_i)$$

Investigation 2: ϕ = additive and time dependent

$$\rho(t) = \lambda + \sum_n C_i t$$

$$R(t) = \exp[-\lambda t] \quad \exp[-\tfrac{1}{2} \sum_n C_i t^2]$$

$$T_f = \exp[\frac{\lambda^2}{2 \sum_n C_i}] \left\{ \sqrt{\frac{\pi}{2 \sum_n C_i}} \left(1 - \text{erf}[\frac{\lambda}{\sqrt{2 \sum_n C_i}}]\right) \right\}$$

See Appendix[B]for proof.

Investigation 3: ϕ = multiplicative and time dependent

$$\rho(t) = (ct)\ \lambda$$

$$R(t) = \exp[-\tfrac{c}{2} \lambda\ t^2]$$

$$T_f = \frac{1.254}{(c\lambda)^{1/2}}$$

Investigation 4: ϕ = additive and a function of a constant and a time dependent term

$$\rho(t) = \lambda + C_1 + C_2 t$$

from correlation with investigation 2

$$R(t) = \exp[-(\lambda+C_1)t] \ \exp[-\tfrac{1}{2} C_2 t^2]$$

$$T_f = \exp[\frac{(\lambda+C)^2}{2C}] \left\{ \sqrt{\frac{\pi}{2C}} \left(1 - \text{erf}[\frac{\lambda+C}{\sqrt{2C}}]\right) \right\}$$

It can be seen that these values for $R(t)$ and T_f can differ greatly from the values obtained for the standard exponential failure distribution, i.e.

$$R(t) = \exp[-\lambda t]$$

$$T_f = \frac{1}{\lambda}$$

Figure XVII shows the relative importance of $\sum_n C_i$ for investigation 2 where $\lambda = .01$.

RESULTS

FIGURE VI

System Reliability vs Time

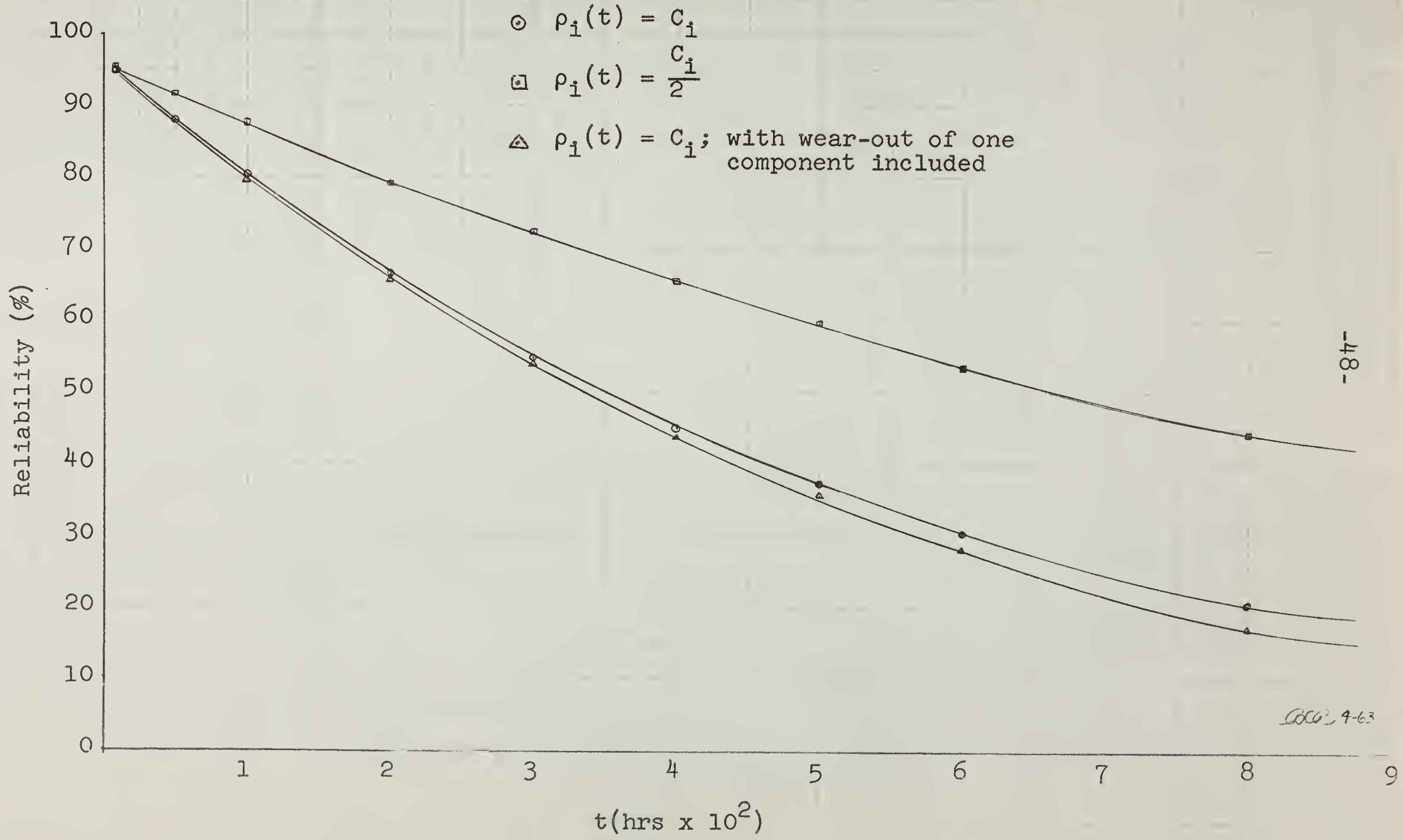


FIGURE VII

Increase in System Reliability Due to a Change in Diesel Engine Instant Failure Rate

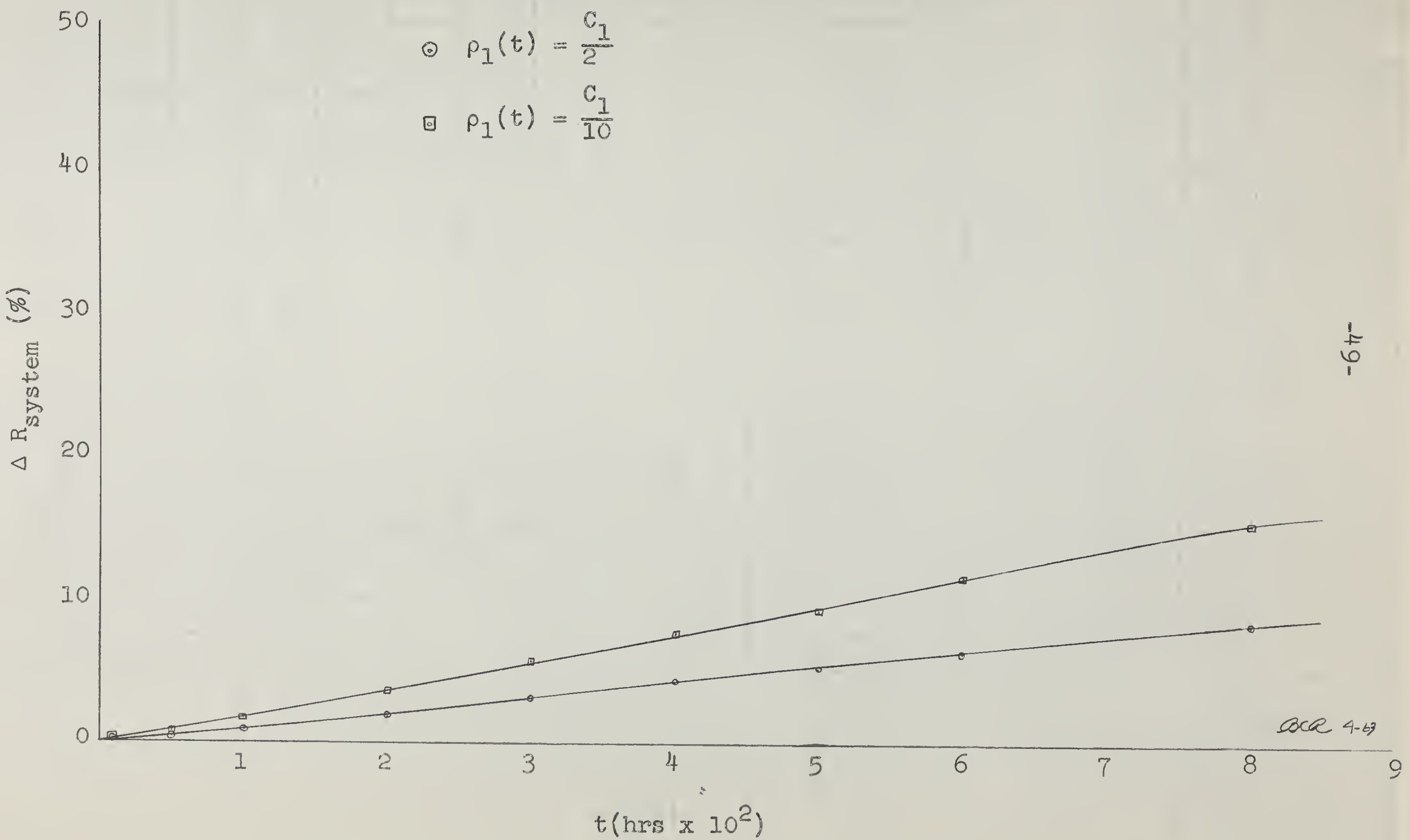


FIGURE VIII

Increase in System Reliability Due to a Change in Gas-Turbine Instant Failure Rate

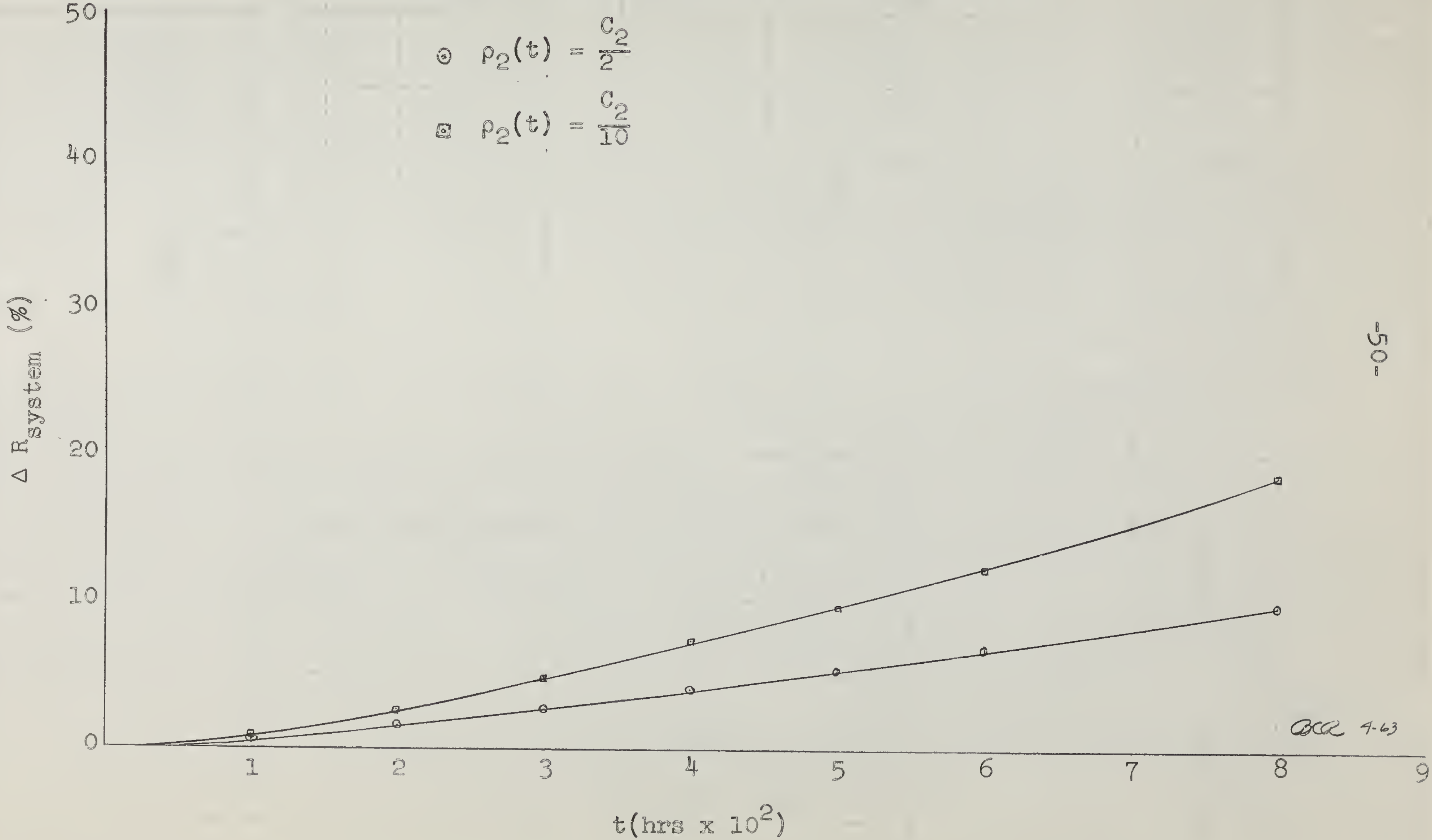


FIGURE IX

Increase in System Reliability Due to a Change in Lube Oil Pump Instant Failure Rate

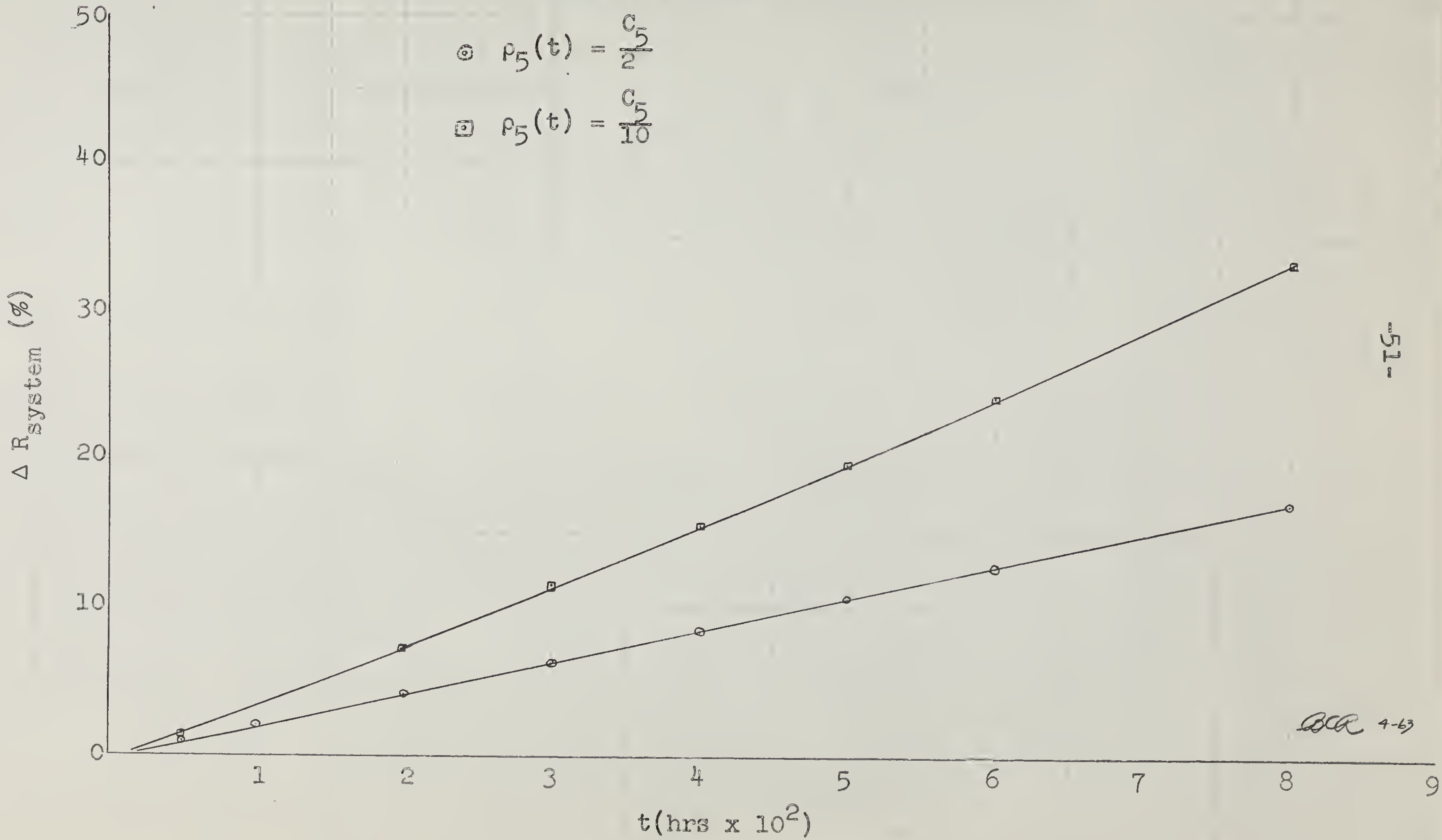


FIGURE X

Increase in System Reliability Due to a Change in Fresh Water Pump Instant Failure Rate

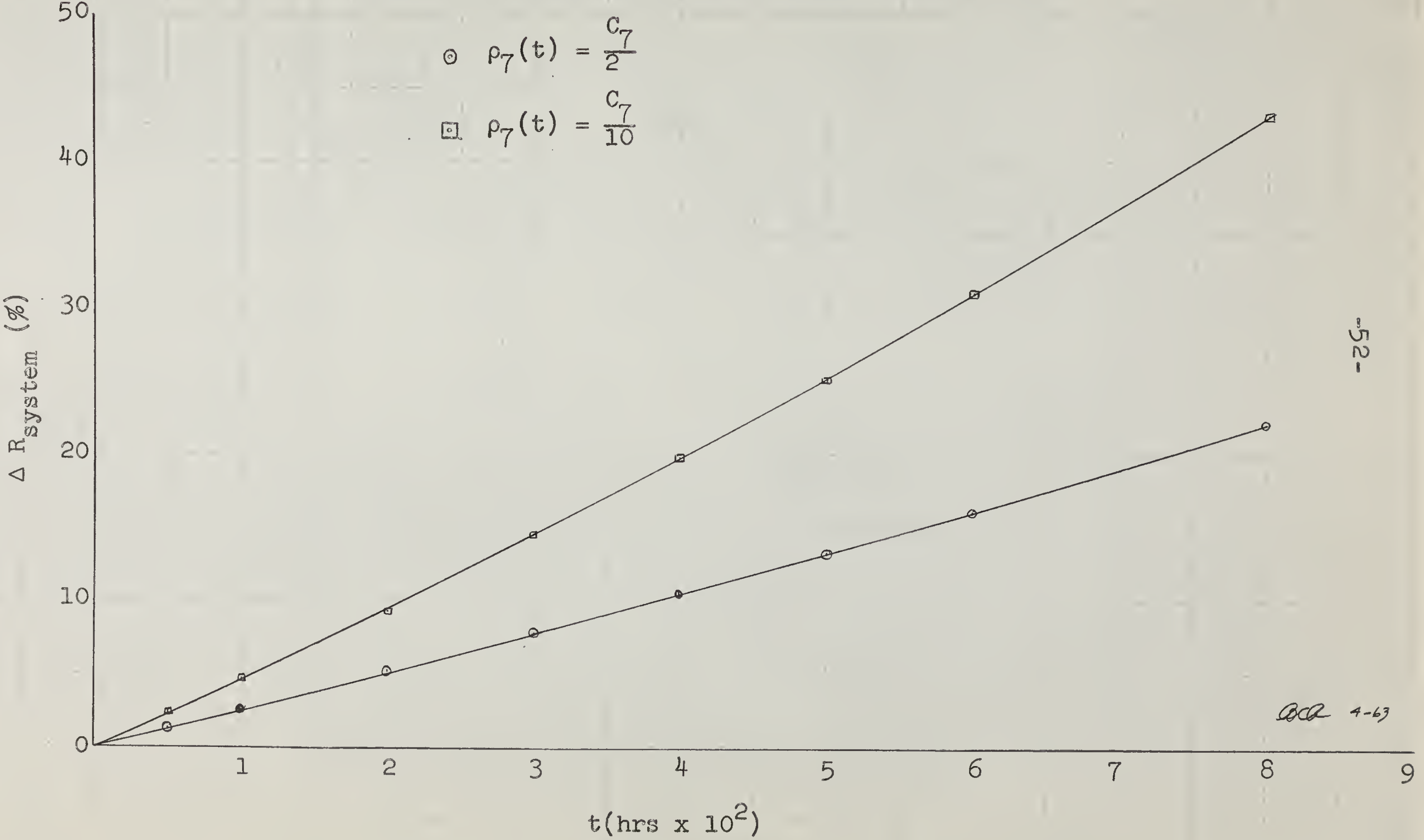


FIGURE XI

Determination of the Policy to Use For
Preventive Maintenance Scheduling

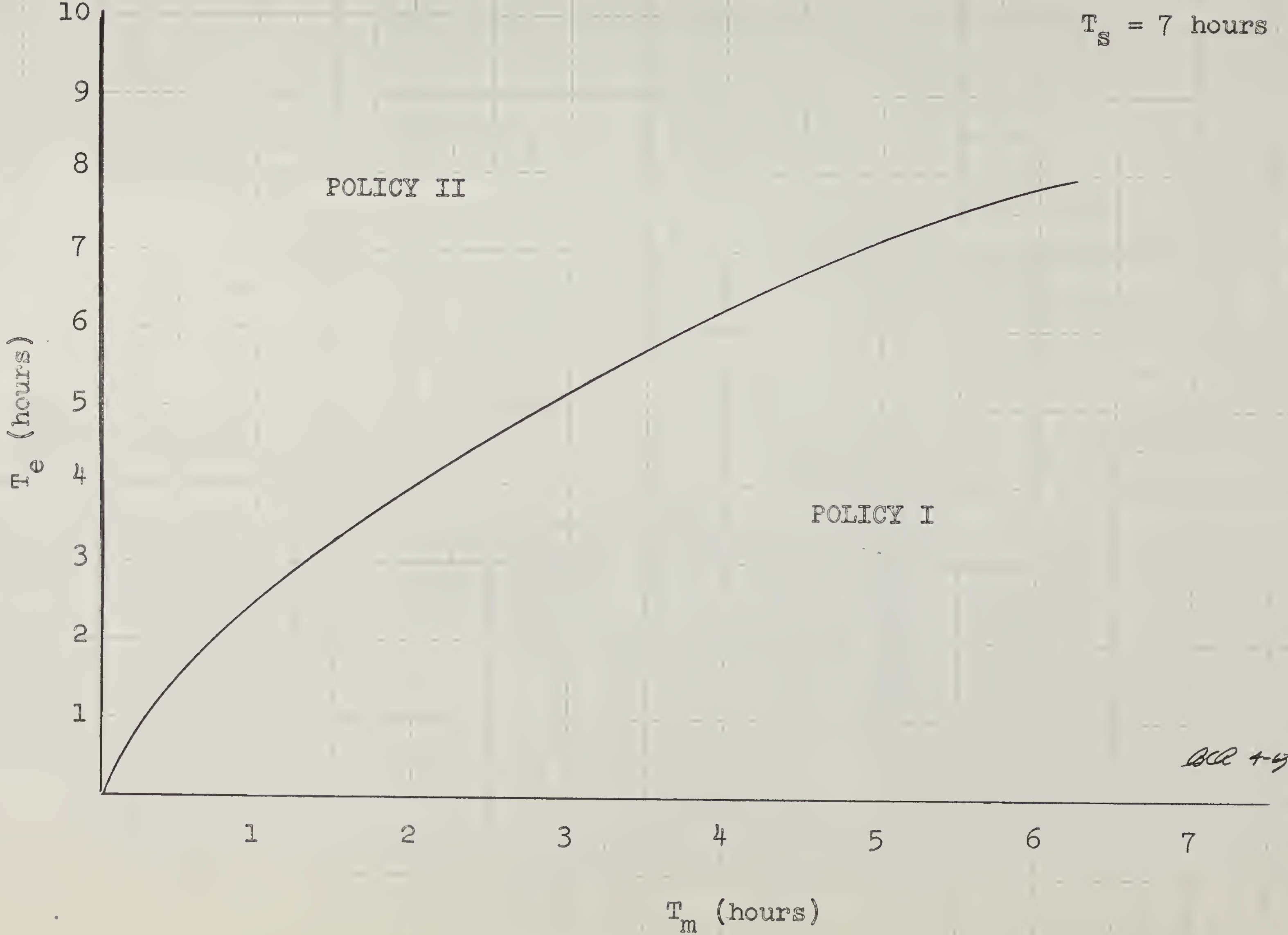


FIGURE XII

Preventive Maintenance Policy by Dynamic Programming for $C_f = 5$

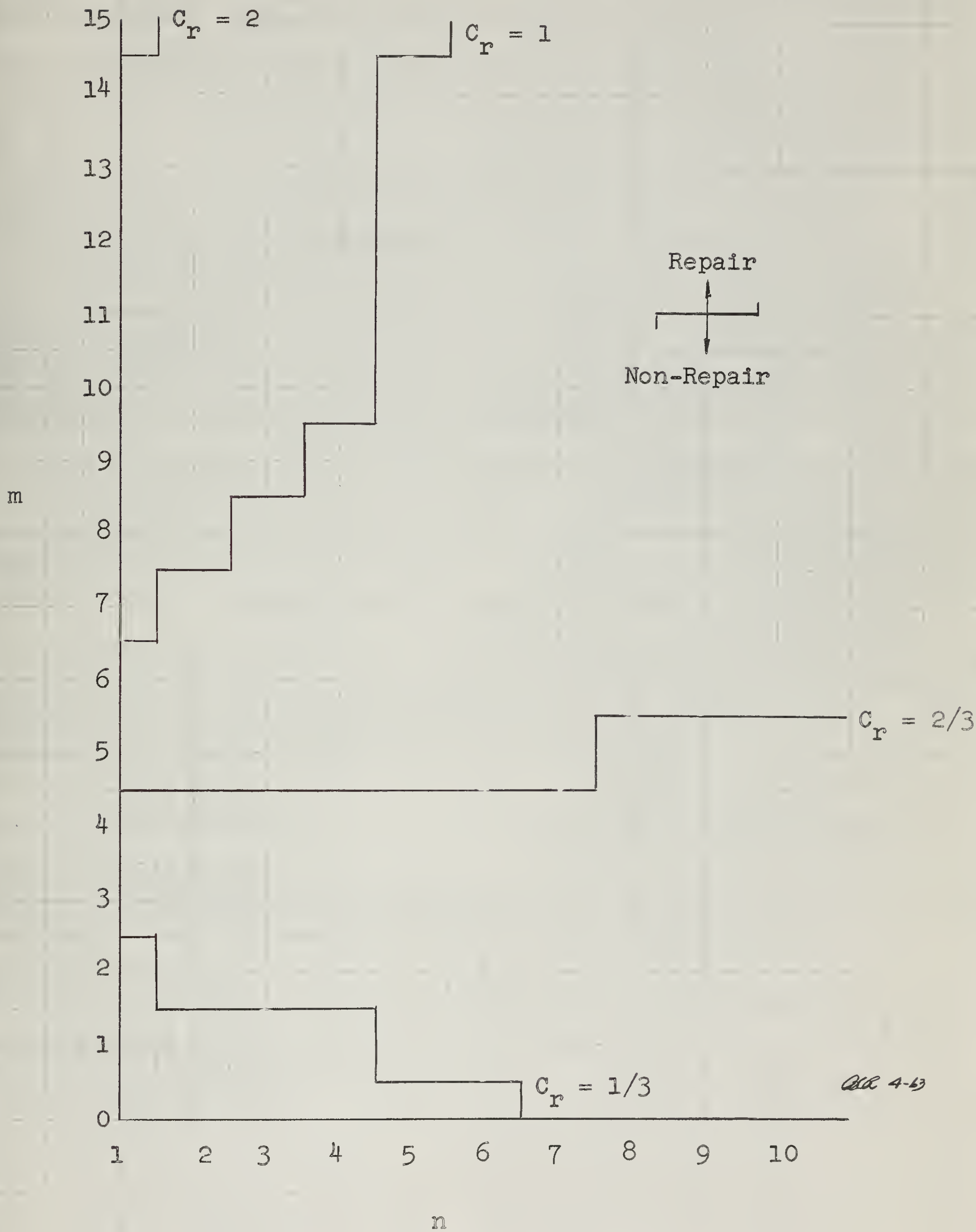


FIGURE XIII

Preventive Maintenance Policy by Dynamic Programming for $C_f = 10$

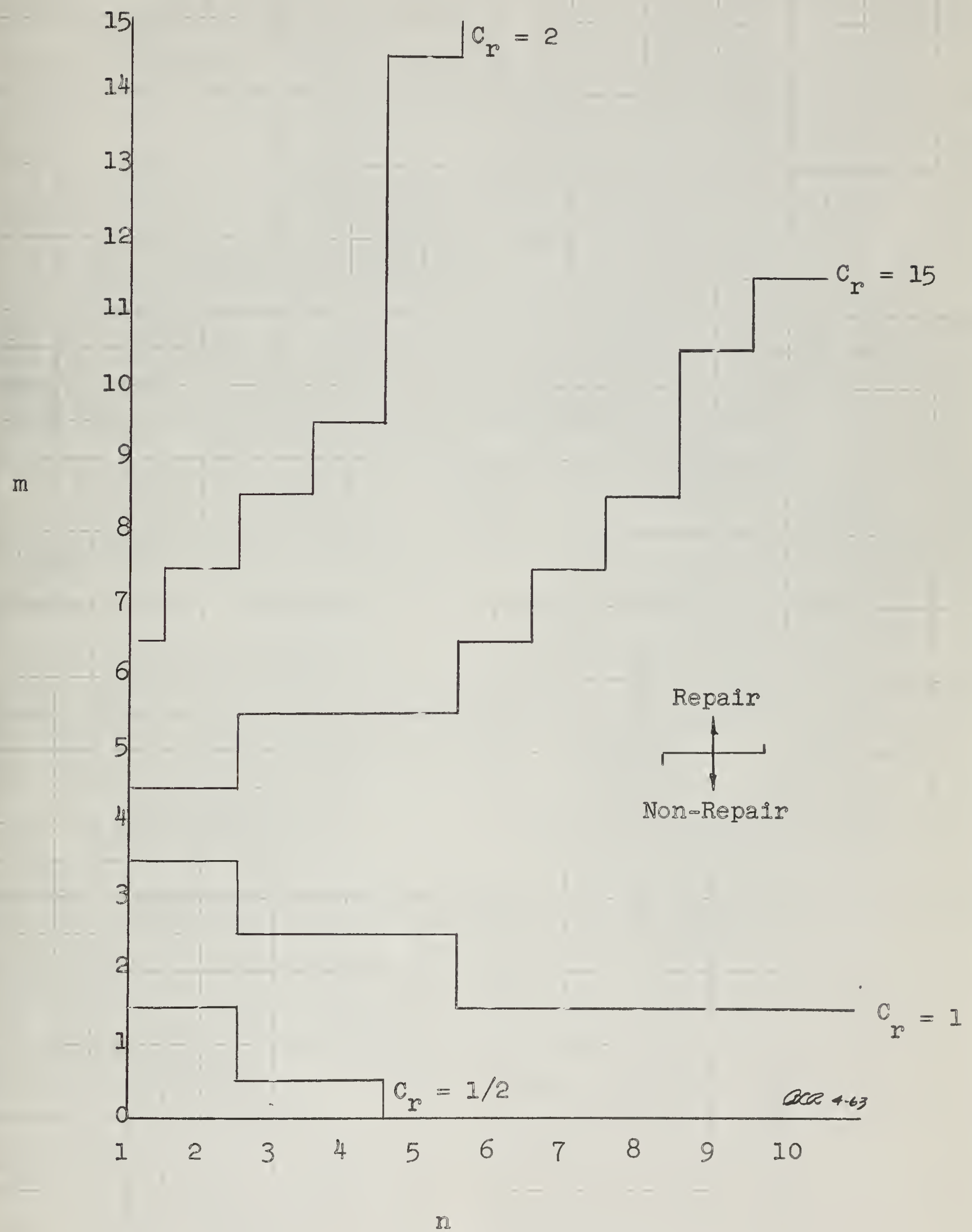


FIGURE XIV

Preventive Maintenance Policy by Dynamic Programming for $C_f = 15$

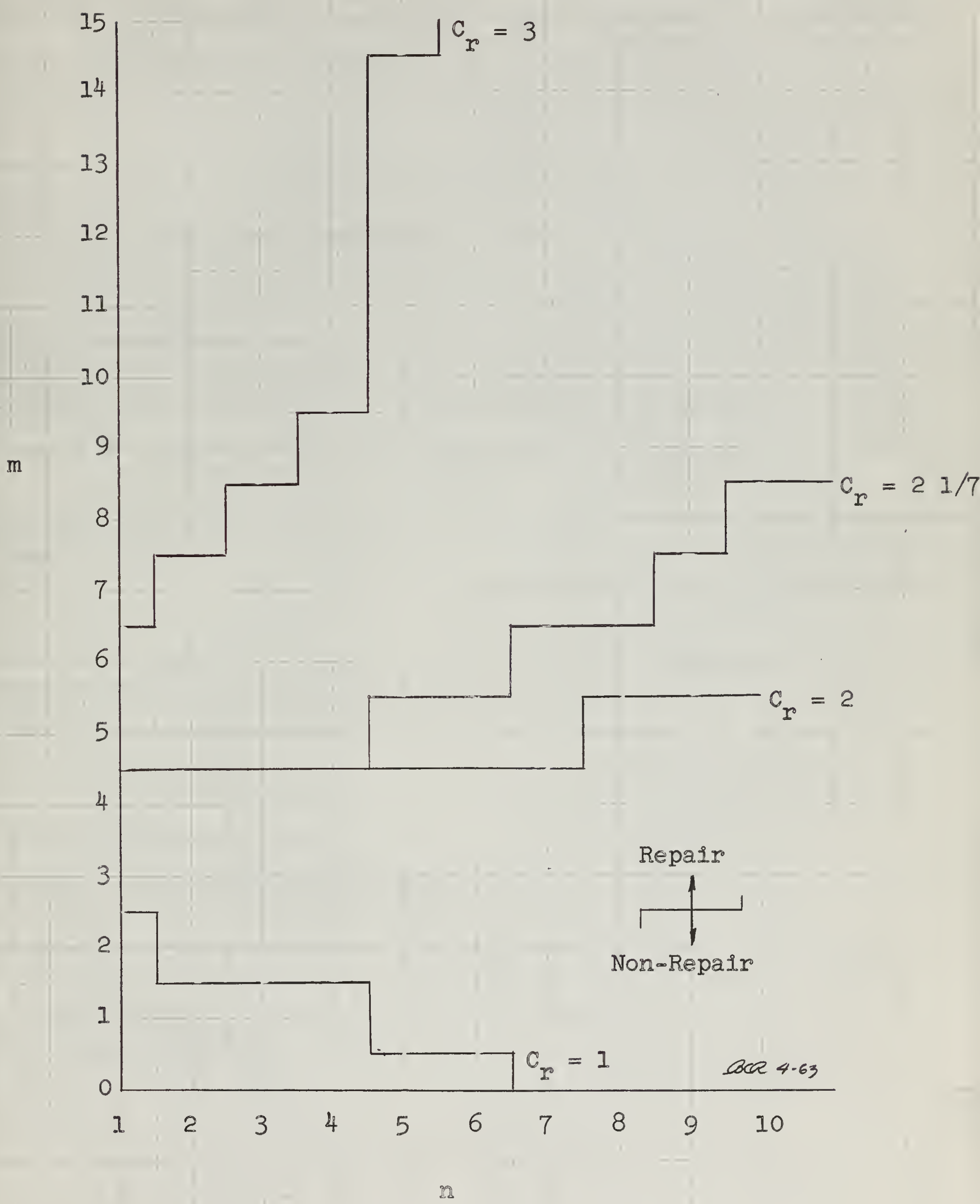


FIGURE XV

Preventive Maintenance Policy by Dynamic Programming for $C_f = 20$

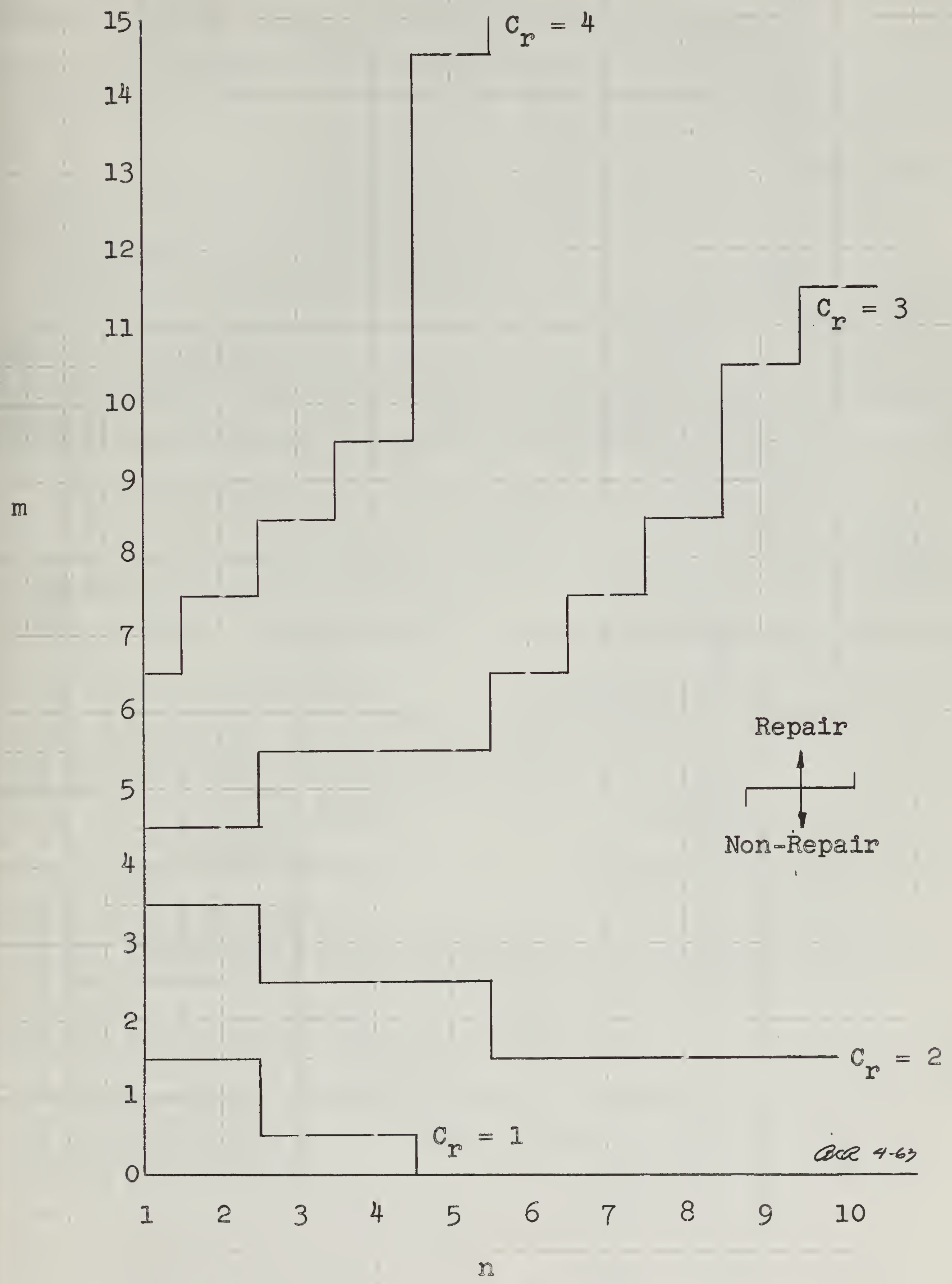


FIGURE XVI

Preventive Maintenance Policy by Dynamic Programming for ratio $\frac{C_f}{C_r}$

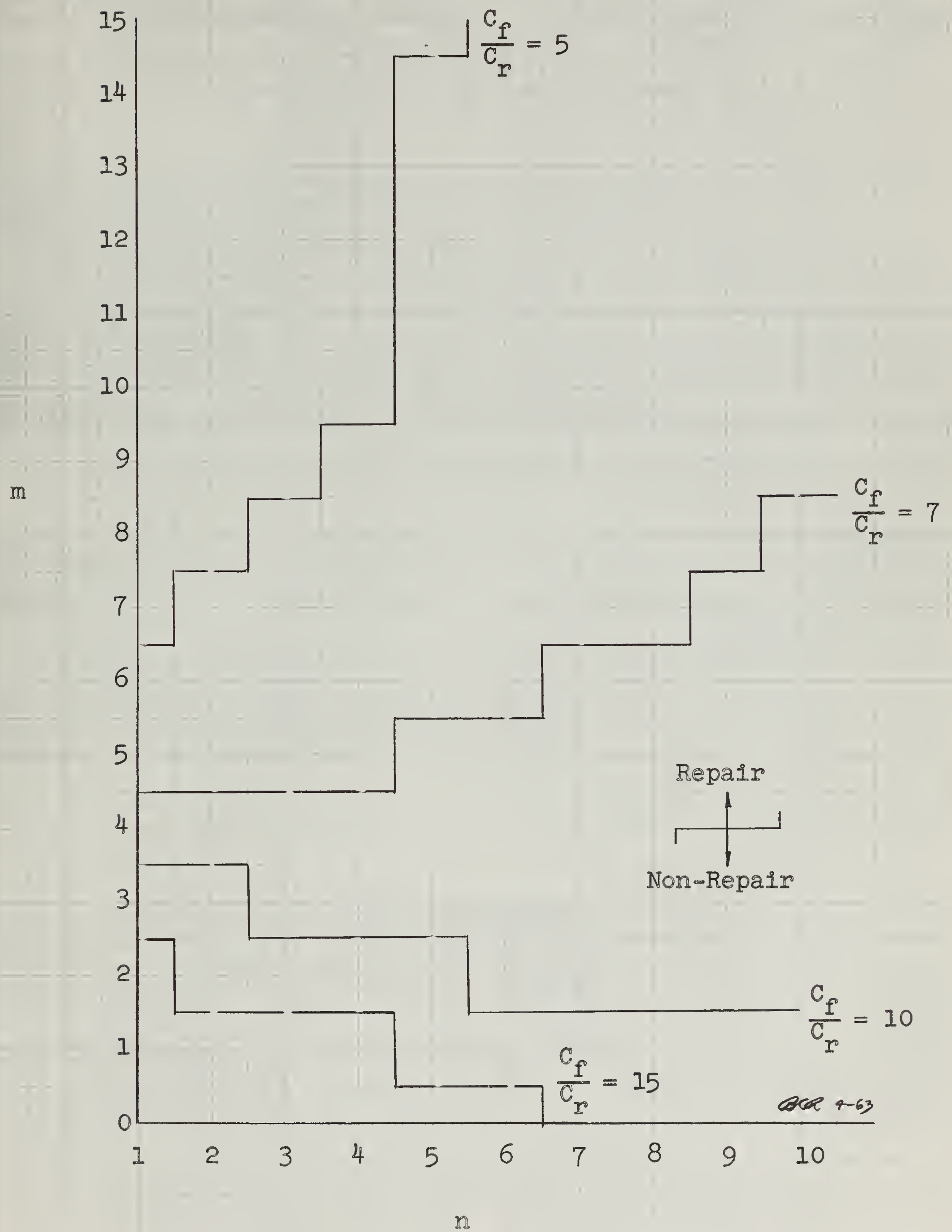
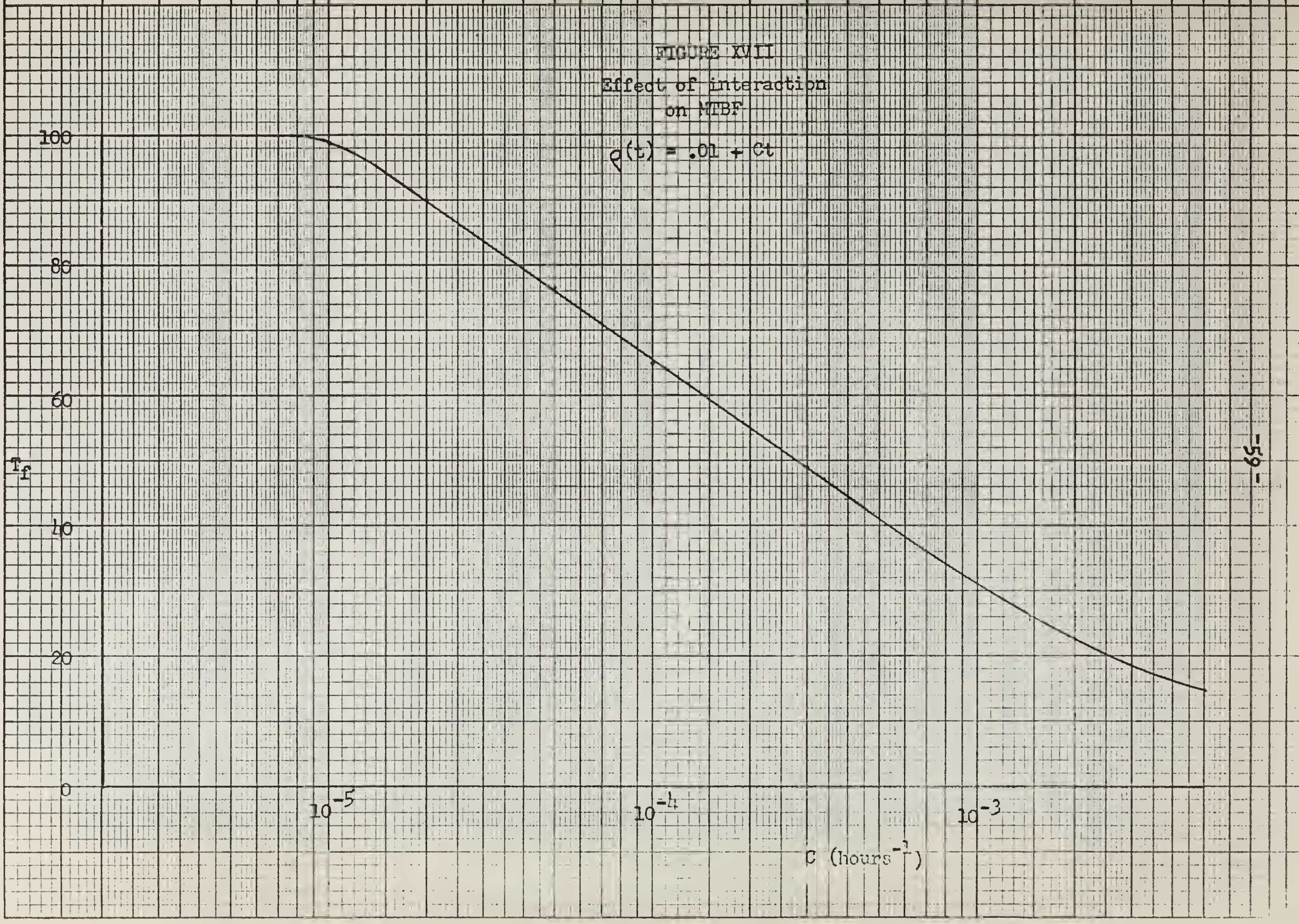


FIGURE XVII

Effect of interaction on MTBF

$\rho(t) = .01 + Ct$



DISCUSSION OF RESULTS

Reliability Analysis

In Figure VI the overall system reliability as a function of time is shown. The plot graphically displays what degree of reliability can be expected from the system assuming;

a. the values for the instant failure rates specified previously.

b. the components are good as new at $t = 0$.

c. no preventive maintenance is performed on the system during the operating cycle.

It can be determined that this overall reliability curve for the system consisting of different components with unlike operating conditions can be approximated by;

$$R_{sys}(t) = e^{-Ct}$$

where $C = .00198$

Other results that can be obtained from Figure VI are;

1. If the instant failure rates of all the components were decreased by 50%, the overall system reliability will be increased greatly as <u>t</u> increases, i.e.

$t = 200$; R_{sys} is increased by 20.2%

$t = 500$; R_{sys} is increased by 61.2%

$t = 800$; R_{sys} is increased by 116.8%

2. If one component has wear-out characteristics (fresh water pump) such that it should be included in the reliability analysis, the overall system reliability is adversely affected. This effect becomes especially prominent as the system

[illegible]

operating time approaches the mean life time of the component and is also dependent upon the varience of the mean life time. Figure VI shows that at t = 800, R_{sys} is decreased by 15.7% due to wear-out. The equation that is applicable when investigating a system or component which exhibits random and wear-out failures during the operating cycle is;

$$R_i(t) = R_{i_w}(t) \cdot R_{i_r}(t)$$

defining $R_i(t)$ = overall reliability of component i

$R_{i_w}(t)$ = reliability of component i exhibiting only wear-out failures

$R_{i_r}(t)$ = reliability of component i exhibiting only random failures

Since the wear-out reliability factor is multiplicative, the system reliability will decrease greatly with time as the number of components experiencing wear-out increases.

3. Figures VII through X illustrate the relative importance of each component to overall system operation from a reliability viewpoint. These figures show percentage-wise how the decrease in a specific components instant failure rate will affect the system by using the formula;

$$R_{sys} = \frac{R_{sys}(\text{with decreased } \rho(t)) - R_{sys}(\rho_i(t)=C_i)}{R_{sys}(\rho_i(t)=C_i)}$$

It is observed that a decrease in the fresh water pumps failure rate by 50% increases the overall system reliability by 22% for an operating cycle of 800 hours while the same decrease for the diesel engines failure rate only increases

the reliability by 7.8%. Therefore if the system reliability needs to be improved, strictly from a reliability standpoint of the four components investigated the fresh water pump performance should be improved first and then the lube oil pump, gas turbine, and diesel respectively. An investigation like this immediately indicates where in the system, improvements should be made to increase overall reliability. Conversely, it also reveals what increase can be expected in system reliability for an improvement in a specific components instant failure rate. Consequently, graphs of the type shown in Figures VI through X, which can be determined for any system if the instant failure rates of its components are known, provide an intimate insight into system behavior and provide valuable information for decisions regarding engineering improvements.

Preventive Maintenance

Figure XI shows the relative merit of using either of the two policies discussed on pages 38 - 40 of this thesis. This graph is representative of the preventive maintenance situation for a system having a failure rate that increases with time. As an example of how to use the figure, assuming $T_s = 7$ hours and the expected value of $T_e = 4$ hours then policy I is applicable unless $T_m \leq 2$ hours. Similarly, for any value of T_e there is a corresponding value of T_m which determines the boundary for choosing the optimum policy to follow. It will generally be found that policy II is most applicable to complex systems

which have many subsystems liable to fail and where keeping records of times to failure for each subsystem becomes cumbersome. It is interesting to note that if costs of failure are substituted for the times of repair, then these same equations minimize the expected cost of operating the system.

The method of determining preventive maintenance scheduling by dynamic programming is more suitable to a marine propulsion plant than the previously discussed method. The reason for this is that with operating schedules to maintain it is probable that optimum preventive maintenance times found will not coincide with the ships schedule. This method also provides a continuous picture of what policy to take regarding preventive maintenance over all combinations of time intervals (trips) completed and time intervals still to be completed. Figures XII through XVI illustrate the results of such an analysis made for a system having the Weibull failure distribution described in the Procedure section. From these graphs the expected cost of maintenance can be minimized for any operational situation of this system if the cost of failure and of maintenance are known. These figures are only applicable for $m, n > 0$ as if $m = 0$, the system has just been repaired and it does no good to repair it again, and if $n = 0$, there are no more trips to make so the expected cost $= \underline{0}$.

Although the graphs are self-explanatory, one interesting observation can be made which is not intuitively obvious nor

[illegible]

would normally be expected if the investigation had not been made. Take as an example, Figure XIII for $C_f = 10$ and the curve representing $C_r = 2$. It is seen that if 8 trips have been made (m = 8) and 2 more are planned (n = 2), the policy should be to repair the system. However, with the same past history and if 3 more trips are necessary (n = 3), the system should not be repaired. The explanation of this is;

1. at large m the probability of failure in n intervals is relatively high and for small n, repair is the best policy since in effect it starts the system off at m = 0.

2. as n increases, this effect of starting at m = 0 becomes less important and the cost of preventive maintenance is the dominant factor. The reason is that at large n, even if maintenance is performed, there is still a significant probability of failure in the following n-1 intervals which causes a greater expected cost than if no repair is made and a high probability of system failure is accepted in the next n intervals.

Analysis of Figures XII through XV provides the results for the determination of Figure XVI. Figure XVI shows that policy determination is solely dependent upon failure distribution, length of interval, and the ratio of C_f to C_r. This fact lessens greatly the number of calculations that have to be made in a system analysis since assuming the system failure distribution and interval time are known, only one graph has to be derived instead of one for each possible value of C_f. As the values of C_f and C_r are bound to

vary with time due to the normal trend of increasing repair and material costs, the advantage of the single graph representing the ratio is evident.

<u>Interacting Components</u>

Although little if any investigation has been made into the problem of the interaction component failure distributions, Figure XVII illustrates that in certain cases this effect can be very important. For instance, it is seen that if $\rho(t) = .01 + 4(10^{-5})t$ the mean time to failure is decreased 20%. On the other hand for $t = \frac{T_f}{5}$, this interaction effect only decreases reliability by less than 1%. Consequently although the effects of interaction may not affect the basic calculations for reliability significantly and therefore may be difficult to observe and determine, they definitely can become significant in the overall system analysis through T_f which is used in many operations analysis calculations not discussed in this thesis, i.e. renewal theory. It therefore becomes imperative that investigations should be made determining any interaction present in a system if worthwhile results are to be obtained from an analysis of the system.

CONCLUSIONS

In addition to the specific conclusions stated in the Discussion of Results, the following general conclusions can be drawn.

1. The operational analysis of a marine propulsion plant will provide results not otherwise available which will be beneficial in making decisions regarding;

   a. preliminary design

   b. operational policies

   c. maintenance actions

   d. system improvement and redesign

2. In such an analysis the definition and formulation of the objectives of the investigation determine all succeeding stages of the analysis and the utility of the results.

3. Statistical data are a necessary prerequisite to any operational analysis. The definition of the problem governs the areas of investigation for the statistical tests.

4. Reliability as a function of time can be characterized by the single parameter $\rho(t)$, instant failure rate. The effect of an increase or decrease in this parameter is amplified as time increases.

5. The wear-out failure distributions of the individual system components should be included in the analysis of a mechanical system as they can greatly affect the overall system reliability. This consequence becomes especially prominent as $\underline{t}$ approaches the MTBF of a component.

6. From an operational analysis of a system, the

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

relationship of each component's performance to overall system operation can be determined. Such information will be of considerable significance in the determination of policies regarding system design and improvement.

7. Preventive maintenance is only advantageous for devices having an instant failure rate that increases with time.

8. For this type device there is an optimal time for the performance of preventive maintenance which will maximize availability. An optimal time may also be found which will minimize expected cost of operation.

9. Dynamic programming can be utilized in determining an optimum maintenance policy for systems operating in discrete time intervals. This policy will give a complete picture at any time in the systems operating cycle of what maintenance action to follow in order to minimize the expected cost of repair.

10. The effect of interacting component failure distributions normally neglected in operational analyses and difficult to determine quantitatively can considerably affect the results of the investigation and should therefore be included.

RECOMMENDATIONS

1. The concept of the operational analysis of marine propulsion plants should be accepted by naval engineers as an important tool in the evaluation of existing and proposed marine systems.

2. A program of component testing and data assimilation should be initiated immediately for the purpose of determining representative failure distributions of mechanical devices.

3. Accepting the fact that corroborating statistical data are minimal, systems analyses should be carried out on existing systems for the object of developing and refining techniques and gaining facility in the procedures of such an investigation.

4. Further examination should be made into the question of how out of tolerence i.e. wear-out failures are to be defined and handled.

5. A detailed investigation should be made into the problem of failure distribution dependence. This should be explored both from a theoretically mathematical and an experimental viewpoint.

6. The applicability of the method of dynamic programming to system analyses questions should be investigated. Possible applications other than for maintenance policies might be;

   a. optimal reliability apportionment among the components.

b. optimal arrangement of components under constraints for maximum system reliability.

c. optimal choice of components to minimize the deleterious effect of failure distribution interaction.

APPENDIX A

Definitions

Failure: a detected cessation of ability to perform a specified function or functions within previously established limits.

Independent Failure: those component failures which occur or can occur without being related to the malfunctioning of associated components.

Redundancy: the existence of more than one means for accomplishing a given task where all means must fail before there is an over-all failure to the system.

Availability: the fraction the total desired operating time that a system is actually operable.

Mean Time Before Failure (MTBF): the mean or average time between successive failures of a component

Cumulative Probability of Failure ($Q(t)$, $F(t)$): probability that the life of a component is less than t.

Probability Density Function of Failure ($f(t)$): probability that the life of a component will be between t and $t + dt$.

Instant Failure Rate ($\rho(t)$): probability that a component will fail during the time interval t to $t + dt$ conditional upon it surviving up to time t.

Reliability $R(t)$: probability that a component will operate within specified limits for the time, t, and operating conditions imposed.

APPENDIX B

Details of Procedure

I. Determination of $R(t)$ for a component used periodically assuming an exponential failure distribution.

$$F(t) = \int_0^t [1-P(\tau)]f(\tau)d\tau + \int_0^t P(\tau)[G(t-\tau)]f(\tau)d\tau$$

$$P(\tau) = P$$

$$G(t-\tau) = 1 - e^{-\frac{(t-\tau)}{N}}$$

$$f(\tau) = C\ e^{-C\tau}$$

$$Q_1(\tau) = \int_0^t [1-P]C\ e^{-C\tau}d\tau = [1-P][1-e^{-Ct}]$$

$$= 1 - P - e^{-Ct} + P\ e^{-Ct}$$

$$Q_2(\tau) = \int_0^t P[1-e^{-\frac{1}{N}(t-\tau)}]C\ e^{-C\tau}d\tau$$

$$= P - P\ e^{-Ct}[1 + \frac{C}{\frac{1}{N} - C}] + P(\frac{C}{\frac{1}{N} - C})e^{-\frac{t}{N}}$$

$$F(t) = 1 - e^{-Ct}[1 + P(\frac{C}{\frac{1}{N} - C})] + P(\frac{C}{\frac{1}{N} - C})e^{-\frac{t}{N}}$$

$$R(t) = 1 - F(t)$$

$$R(t) = e^{-Ct}[1 + P(\frac{C}{\frac{1}{N} - C})] - P(\frac{C}{\frac{1}{N} - C})e^{-\frac{t}{N}}$$

II. Determination of $R(t)$ for a standby system assuming exponential failure distributions.

$$\rho(t) = C_1 \quad \text{for main component}$$

$$\rho(t) = C_2 \quad \text{for standby component}$$

For a standby system $Q(t) = \int_0^t F_2(t-t_1)f_1(t_1)dt_1$

$$F_2(t-t_1) = \int_{t_1}^{t} f(t-t_1)dt = \int_{t_1}^{t} C_2 e^{-C_2(t-t_1)} dt$$

$$= 1 - e^{-C_1(t-t_1)}$$

$$Q(t) = \int_{o}^{t} F_2(t-t_1) f_1(t) dt_1 = \int_{o}^{t} [1-e^{-C_2(t-t_1)}] C_1 e^{-C_1 t_1} dt_1$$

$$= \int_{o}^{t} C_1 e^{-C_1 t_1} dt_1 - C_1 e^{-C_2 t} \int_{o}^{t} e^{(C_2-C_1)t_1} dt_1$$

$$= 1 - e^{-C_1 t} - \frac{C_1}{C_2-C_1}(e^{-C_2 t})[e^{(C_2-C_1)t_1}]_{o}^{t}$$

$$= 1 - e^{-C_1 t} - \frac{C_1}{C_2-C_1}[e^{-C_1 t} - e^{-C_2 t}]$$

$$R(t) = 1 - Q(t)$$

$$R(t) = e^{-C_1 t}[1 + \frac{C_1}{C_2-C_1}] - e^{-C_2 t}$$

III. Determination of $R(t)$ and T_f for $\rho(t) = \lambda + C_1 t$

$$\rho(t) = \lambda + C_1 t$$

$$R(t) = \exp[-\int_{o}^{t}(\lambda + C_1\tau)d\tau]$$

$$= \exp[-\int_{o}^{t} \lambda d\tau - \int_{o}^{t} C_1\tau \, d\tau]$$

$$= \exp[-\lambda t - C_1 \int_{o}^{t} \tau \, d\tau]$$

$$= \exp[-\lambda t - \frac{C_1}{2} t^2]$$

$$= e^{-\lambda t} e^{-\frac{1}{2} C_1 t^2}$$

$$T_f = \int_{o}^{\infty} R(t)dt$$

$$= \int_0^\infty \exp[-(\lambda t + \frac{1}{2} C_1 t^2)]dt$$

From C.R.C. Standard Mathematical Tables [7]

$$\int_0^\infty \exp[-a^2x^2]dx = \frac{1}{2a}\sqrt{\pi} = \frac{.887}{a}$$

it can be seen that

$$\frac{1}{2} C_1 t^2 + \lambda t = \frac{1}{2} C_1 [t^2 + \frac{2\lambda}{C_1} t (+ \frac{\lambda}{C_1}^2 - \frac{\lambda}{C_1}^2)]$$

$$= \frac{1}{2} C_1 [(t + \frac{\lambda}{C_1})^2 - (\frac{\lambda}{C_1})^2]$$

$$= \frac{1}{2} C_1 (t + \frac{\lambda}{C_1})^2 - \frac{\lambda^2}{2\ C_1}$$

therefore

$$R(t) = \exp[\frac{\lambda^2}{2\ C_1}] \exp[-\frac{1}{2} C_1 (t + \frac{\lambda}{C_1})^2]$$

therefore

$$T_f = \int_0^\infty R(t)dt = \exp[\frac{\lambda^2}{2\ C_1}] \int_0^\infty \exp[-\frac{1}{2} C_1 (t + \frac{\lambda}{C_1})^2]dt$$

let $x = t + \frac{\lambda}{C_1}$ $\quad t = 0 \quad x = \frac{\lambda}{C}$

$dx = dt$ $\quad t = \infty \quad x = \infty$

$$\int_0^\infty \exp[-\frac{1}{2} C_1 (t + \frac{\lambda}{C_1})^2]dt = \int_{\frac{\lambda}{C}}^\infty \exp[-\frac{1}{2} C_1 (x^2)]dx$$

which equals (dropping the subscript)

$$\int_0^\infty \exp[\frac{C}{2} x^2]dx - \int_0^{\frac{\lambda}{C}} \exp[\frac{C}{2} x^2]dx$$

from C.R.C. tables

$$\int_0^\infty \exp[\frac{C}{2} x^2]dx = \sqrt{\frac{\pi}{2C}}$$

now let $t = \sqrt{\frac{C}{2}}\, x \qquad x = 0 \qquad t = 0$

$$dt = \sqrt{\frac{C}{2}}\, dx \qquad x = \frac{\lambda}{C} \qquad t = \frac{\lambda}{\sqrt{2C}}$$

therefore

$$\int_0^{\frac{\lambda}{C}} \exp[\tfrac{C}{2}\, x^2]dx = \left(\frac{2}{\sqrt{\pi}}\right)\left(\frac{\sqrt{\pi}}{2}\right) \sqrt{\frac{2}{C}} \int_0^{\frac{\lambda}{\sqrt{2C}}} \exp[-t^2]dt$$

and

$$\frac{2}{\sqrt{\pi}} \int_0^{\frac{\lambda}{\sqrt{2C}}} \exp[-t^2]dt = \text{erf}[\frac{\lambda}{\sqrt{2C}}]$$

thus

$$\int_0^{\infty} \exp[\tfrac{C}{2}\, x^2)dx = \sqrt{\frac{\pi}{2C}}\, [1 - \text{erf}(\frac{\lambda}{\sqrt{2C}})]$$

and

$$T_f = \exp[\frac{\lambda^2}{2C}]\ \sqrt{\frac{\pi}{2C}}\, (1 - \text{erf}[\frac{\lambda}{\sqrt{2C}}])$$

IV <u>Main Body of Computer Program for Solution of Equation</u>

(FORTRAN)

```
      DO  114     N = 1, N1
      JJ = 0
      DO  114     M = 1, M1
      IF(N-1)   100, 100, 101
100   A1 = 0.
      GO TO 102
101   J = N - 1
      A1 = COST1(J,1)
102   A2 = CASREP + A1
      A3 = RATE(M) * A2
      IF(N-1) 103, 103, 104
103   A4 = 0.
      GO TO 105
104   K = M + 1
      A4 = COST1(J,K)
105   A5 = 1.-RATE(M)
      A6 = A5 * A4
      COST1(N,M) = A3 + A6
      IF(N-1) 106, 106, 107
106   B1 = 0.
      GO TO 108
107   B1 = COST2(J,1)
108   B2 = SKDREP + RATE(1) * (CASREP + B1)
      IF(N-1) 109, 109, 110
109   B3 = 0.
```

```
      GO TO 111
110   B3 = COST2(J,2)
111   B4 = 1.-RATE(1)
      B5 = B4 * B3
      COST2(N,M) = B5 + B2
      IF(JJ) 112, 112, 113
112   JJ = 1
      PRINT 20
      PRINT 21, N
113   L = M - 1
114   PRINT 22, L, COST1(N,M), L, COST2(N,M)
```

APPENDIX C

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